Efficient Sum-Rate Maximization for Medium-Scale MIMO AF-Relay Networks

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Abstract

We consider the problem of sum-rate maximization in multiple-input multiple-output (MIMO) amplify-and-forward (AF) relay networks with multi-operator. The aim is to design the MIMO relay amplification matrix (i.e., the *relay beamformer*) to maximize the achievable communication sum-rate through the relay. The design problem for the case of single-antenna users can be cast as a non-convex optimization problem, which in general, belongs to a class of NP-hard problems. We devise a method based on the minorization-maximization technique to obtain quality solutions to the problem. Each iteration of the proposed method consists of solving a strictly convex unconstrained quadratic program; this task can be done quite efficiently such that the suggested algorithm can handle the beamformer design for relays with up to ~ 70 antennas within a few minutes on an ordinary personal computer (PC). Such a performance lays the ground for the proposed method to be employed in medium-scale (or lower-regime massive) MIMO scenarios.

Keywords: Amplify and forward, beamforming, medium-scale MIMO, minorization-maximization (majorization-minimization), massive MIMO, relay networks, sum-rate

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I. INTRODUCTION

Sum-rate maximization is a fundamental task arising in signal design for communication, and particularly relay networks, in which relays are often used to enhance the quality of communication between pairs of users within the network. In such networks, two-way relaying is shown to achieve better spectral efficiency as compared to one-way relaying [1]— a fact that has attracted more research interest to two-way relay networks and in studying them from both theoretical and practical points of view.

Note that the rate-optimal strategy for two-way relaying is not yet known in general scenarios [1]–[3], particularly if one considers the case of several communication operators that provide communication services in the network (referred to as operator in the sequel). However, various protocols including decode-and-forward (DF), and amplify-and-forward (AF) have been proposed in the literature for two-way relay networks [4], [5]. Contrary to the DF case, the AF relaying does not perform any signal decoding at the relay, and hence enjoys a lower hardware and software complexity, as well as smaller transmission delay. Interestingly, such simple processing of AF relying is key to large-scale multiple-input multiple-output (MIMO) systems. In practice, relays can be equipped with multiple antennas for performance improvement which leads to a *MIMO relaying* scheme.

Fig. 1 illustrates a schematic of MIMO multi-operator two-way relay networks (with a large number of antennas for the relay). User pairs from different operators employ the AF relay (with a large number of antennas) to achieve a better quality of communication. Note that the mathematical formulation of the illustrated scenario is the same as multi-pair two-way AF relaying. Multi-pair two-way relaying is a generalization of two-way relaying in which more than one pair of nodes exchange information within the network, by employing the shared relay. Since these pairs work in the same time slots and frequency bands, multi-pair two-way relaying improves the spectral efficiency, but it requires further processing to cancel inter-pair interference. Note that these systems have several potential practical applications, e.g. when multiple communication partners (belonging to different operators) use one relay (possibly owned by a third party/virtual operator) and in the same time/spectrum (see also [6] and references therein for more details/examples).

The sum-rate of a MIMO AF relay system depends on the amplification matrix, i.e. the

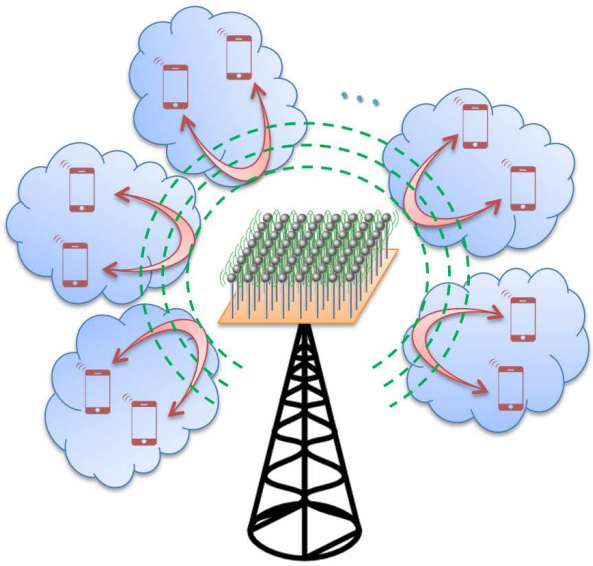


Fig. 1. A schematic of MIMO multi-operator two-way relay networks. User pairs from different operators employ the relay (with a large number of antennas) to achieve a better quality of communication.

beamformer of the relay. Therefore, the aim of several works is to design the relay amplification matrix to maximize the sum-rate of the network. The sum-rate maximization problem for the above mentioned networks is a non-convex optimization problem (and belong to a class of NP-hard problems [1]) and several algorithms have been proposed in the literature to deal with the aforementioned problem. In [7], a branch-and-bound method was employed to tackle this design problem in the single-operator case resulting in an overwhelming[1] computational burden. A related method, i.e., the polyblock approximation algorithm [9], was applied in a similar scenario considering monotonic optimization that can only be used as a benchmark for small/medium MIMO relay networks due to a prohibitive computational complexity in case of a large number of antennas (see also [10]). The authors of [1] developed a polynomial-time iterative method based on a semidefinite relaxation (referred to as POTDC[2]) to tackle the problem. POTDC guarantees a rank-one solution only for the special case of single operator and hence, its solution is generally associated with a synthesis loss. Furthermore, each iteration of POTDC consists of solving a convex determinat maximization (MAXDET) optimization that has a large computational burden.

[1]The branch-and-bound algorithm generally has an exponential computational cost [8].

[2]**PO**lynomial-**T**ime algorithm for **D**ifference of **C**onvex programming.

On the other hand, POTDC results outperform those obtained by the approximate (projection-based) algorithm suggested in [6]. The references [1] and [11] include two heuristic algorithms based on one and two dimensional searches for the special case of single operator. Suboptimal methods based on zero-forcing (ZF) and maximum-ratio combining (MRC) have been presented in [12] for the case of one operator. This work also includes an iterative algorithm solving a semidefinite program at each iteration. The interested reader can refer to [9], [13], [14], [15] and [16] for other approximate or heuristic approaches devised to tackle the sum-rate maximization problem.

In the case of arbitrary number of operators, there is no efficient method that can lead to (some type of) optimality of the obtained solution. The heuristic methods are mainly based on observations for special cases and structured channels. Furthermore, most of the proposed methods in the literature are merely suitable for small scale problems (see e.g. [1], [13]).

The large-scale MIMO concept addresses employing a large number of antennas for transmit/receive leading to superior performance improvements for the systems when compared to ordinary MIMO systems [17]–[19]. In particular, for sum-rate maximization, it has been shown that the ZF and MRC are nearly optimal for very large-scale (i.e., massive) MIMO systems; viz., when the number of antennas diverges to infinity under certain conditions[3] (see e.g., [2], [20], [22]). However, there exist systems with moderate or even relatively large number of antennas for which the asymptotic results do not hold; indeed, how large the number of antennas should be depends on the scenario. Note that conditions for near optimality of ZF/MRC are not satisfied in medium-scale (i.e., lower-regime massive) MIMO scenarios generally. Therefore, for these MIMO systems beamformer design for sum-rate maximization will be required to improve quality of service.

In light of the above, the main contributions of this work can be summarized as follows:

- The problem is considered in a rather general form enabling the user to freely choose the number of operators and the structure of the associated matrices (i.e., the channel parameters).

[3]For example, in [2] it has been shown that the ratio $\frac{number\ of\ antennas}{number\ of\ users}$ should diverge to infinity for the asymptotic results to hold. Furthermore, an analysis of the behavior of the sum-rate for the systems with respect to the number of antennas at the relay and the number of users has been considered in [20] (see also [21] for a related problem).

- We devise an iterative method based on the minorization-maximization technique to tackle the design problem. Applying the proposed method increases the value of the objective function (i.e., the sum-rate) at each iteration. Therefore, it can be shown that the obtained solution is a stationary point of the problem (under some mild conditions, see [23] and references therein) satisfying the first-order optimality criterion for arbitrary number of operators. It is worth mentioning that the general case with arbitrary number of operators leads to a more difficult optimization problem—particularly, the current methods based on semidefinite relaxation can be applied only in the case of single operator (see the discussion before Lemma 1).
- The proposed method is computationally efficient and hence can be applied to medium-scale (or lower-regime massive) MIMO systems as well. Indeed, each iteration of the devised method consists of solving a convex unconstrained quadratic programme (QP); which can be efficiently done for instance with an $\mathcal{O}(n^{2.3})$ complexity (where n is the problem dimension given by square of the number of antennas) [24]. As a result, the method can handle problems with $n \sim 10^3$ variables (or equivalently $M_R \sim 70$) on an ordinary personal computer (PC) within a few minutes.

The rest of this paper is organized as follows. In Section II, we present the system model and problem formulation. We propose an algorithm for designing the beamformer matrix in Section III, followed by several observations in Section IV. Section V includes several numerical examples. Through an efficiency investigation, we show that the proposed method can be applied to relays with large MIMO arrays. Finally, conclusions are drawn in Section VI.

Notation: We use bold lowercase letters for vectors/sequences and bold uppercase letters for matrices. See Table I for other notations used throughout this paper.

II. Problem Formulation

We consider a MIMO AF two-way relay network consisting of M_R antennas, L (communication) operators and pairs of user terminals belonging to different operators (see the discussion of Fig. 1 in Section I and [6] along with references therein for details/practical applications of such systems). We assume single-antenna user terminals and flat fading channels between the k^{th} user of the l^{th} operator and the relay, which are denoted by $\{\mathbf{h}_{k,l}\}$ [1]. The received signal

TABLE I

NOTATIONS

$x(k)$:	the k^{th} entry of the vector $\mathbf{x}$
$\|\mathbf{x}\|_n$:	the l_n-norm of the vector $\mathbf{x}$, defined as $\left(\sum_k \|x(k)\|^n\right)^{\frac{1}{n}}$
$\mathbf{X}^H$:	the transpose conjugate of a matrix $\mathbf{X}$
$\mathbf{X}^T$:	the transpose of a matrix $\mathbf{X}$
tr($\mathbf{X}$):	the trace of a matrix $\mathbf{X}$
$\lambda_{max}(\mathbf{X})$:	the maximum eigenvalue of the hermitian matrix $\mathbf{X}$
$\lambda_n(\mathbf{X})$:	the n^{th} eigenvalue of the matrix $\mathbf{X}$
$\|\mathbf{X}\|_F$:	the Frobenius norm of a matrix $\mathbf{X}$
$\mathbf{X} \succeq \mathbf{Y}$:	$\mathbf{X} - \mathbf{Y}$ is positive semidefinite
$\otimes$:	the Kronecker product
vec($\mathbf{X}$):	the vector obtained by column-wise stacking of $\mathbf{X}$
$\mathbf{I}_n$:	the identity matrix of $\mathbb{C}^{n\times n}$
$\mathbf{e}_l^{(n)}$:	the l^{th} standard basis vector in $\mathbb{R}^n$
$\mathbb{R}$:	the set of real numbers
$\mathbb{C}$:	the set of complex numbers
$\Re(x)$:	the real part of x
$\nabla f(\mathbf{x})$:	the gradient of the differentiable function $f(\mathbf{x})$
$\nabla^2 f(\mathbf{x})$:	the Hessian of the two-times differentiable function $f(\mathbf{x})$
$f(n) = \mathcal{O}(n^x)$:	$f(n)$ is upper bounded by $c\,n^x$ for some $0 < c < \infty$
$\mathrm{E}\{x\}$:	the expected value of the random variable x
$\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$:	the zero-mean random vector $\mathbf{x}$ is distributed according to the circularly symmetric complex Gaussian distribution with covariance $\mathbf{\Sigma}$.

at the relay can be expressed as [1], [6],

$$\mathbf{r} = \sum_{l=1}^{L} \sum_{k=1}^{2} \mathbf{h}_{k,l} x_{k,l} + \mathbf{n}_R \tag{1}$$

where $x_{k,l}$ is the transmitted symbol by the k^{th} user of the l^{th} operator with power $p_{k,l}$ (given by $\mathrm{E}\{|x_{k,l}|^2\}$), and $\mathbf{n}_R$ denotes the circularly symmetric white Gaussian noise with covariance matrix $\sigma_R^2\mathbf{I}$ at the relay. By employing the AF protocol, the transmit signal of the relay is given by

$$\widetilde{\mathbf{r}} = \mathbf{G}\mathbf{r} \tag{2}$$

with $\mathbf{G} \in \mathbb{C}^{M_R \times M_R}$ being the relay amplification matrix, which is to be designed. We assume reciprocal channels between the relay and users [6]; consequently, the received signal $y_{k,l}$ of the k^{th} user at the l^{th} operator becomes

$$y_{k,l} = \mathbf{h}_{k,l}^T \widetilde{\mathbf{r}} + n_{k,l} \tag{3}$$

where $n_{k,l}$ is the associated (white) noise component (with variance $\sigma_{k,l}^2$). The sum-rate of the system can be formulated as [6]

$$R_{sum} = \frac{1}{2} \sum_{l=1}^{L} \sum_{k=1}^{2} \log_2(1 + \eta_{k,l}). \tag{4}$$

Herein $\eta_{k,l}$ denotes the signal-to-interference-plus-noise ratio (SINR) for the k^{th} user of the l^{th} operator and it has the following expression (see Appendix A for the detailed derivation of this expression)

$$\eta_{k,l} = \frac{\mathbf{g}^{\mathrm{H}} \mathbf{\Phi}_{k,l} \mathbf{g}}{\mathbf{g}^{\mathrm{H}} (\mathbf{\Upsilon}_{k,l} + \mathbf{\Delta}_{k,l}) \mathbf{g} + \sigma_{k,l}^2} \tag{5}$$

where $\mathbf{g} = \mathrm{vec}(\mathbf{G})$ and the matrices $\mathbf{\Phi}_{k,l}, \mathbf{\Upsilon}_{k,l}, \mathbf{\Delta}_{k,l}$ are defined as

$$\begin{aligned} \mathbf{\Phi}_{k,l} &= p_{k,l} \left(\mathbf{h}_{3-k,l}^T \otimes \mathbf{h}_{k,l}^T \right)^H \left(\mathbf{h}_{3-k,l}^T \otimes \mathbf{h}_{k,l}^T \right) \\ \mathbf{\Upsilon}_{k,l} &= \sum_{\widetilde{k}} \sum_{\widetilde{l} \neq l} p_{\widetilde{k},\widetilde{l}} \left(\mathbf{h}_{\widetilde{k},\widetilde{l}}^T \otimes \mathbf{h}_{k,l}^T \right)^H \left(\mathbf{h}_{\widetilde{k},\widetilde{l}}^T \otimes \mathbf{h}_{k,l}^T \right) \\ \mathbf{\Delta}_{k,l} &= \sigma_R^2 \left(\mathbf{I}_{M_R} \otimes (\mathbf{h}_{k,l} \mathbf{h}_{k,l}^H)^T \right). \end{aligned} \tag{6}$$

Note that in the definition of the SINR in (5) and (6), the effect of the "self-interference" has been ignored—due to the assumption that terms corresponding to self-interference can be canceled using the channel knowledge[4] (see e.g., [28] and [6] for details). The devised method in this paper, however, can also be applied to the sum-rate maximization problem without making such an assumption.

[4]The channel state information (CSI) of all links is required at the relay which can be estimated if each user sends a training block of length $N_t \geq 2L$ to the relay [26]. Furthermore, the considered time-division duplex (TDD) relaying leads to reciprocal channels between the users and the relay [27]. Consequently, the downlink channel matrix can be obtained by transposing the uplink one and taking calibration into account. At the users side, the k^{th} user of l^{th} operator should only know two scalar parameters $\mathbf{h}_{k,l}^T \mathbf{G} \mathbf{h}_{k,l}$ and $\mathbf{h}_{k,l}^T \mathbf{G} \mathbf{h}_{3-k,l}$ for self-interference cancellation as well as data detection, respectively (that may be estimated by forwarding the training signal received by the relay).

The sum-rate maximization is constrained via the total available power P_R at the relay, viz.

$$\begin{aligned} \mathrm{E}\{\|\widetilde{\mathbf{r}}\|_2^2\} &= \mathrm{tr}\{\mathrm{E}\{\mathbf{G}\mathbf{r}\mathbf{r}^H\mathbf{G}^H\}\} \\ &= \sum_{l=1}^{L}\sum_{k=1}^{2} p_{k,l}\|\mathbf{G}\mathbf{h}_{k,l}\|_2^2 + \sigma_R^2\|\mathbf{G}\|_F^2 \leq P_R \end{aligned} \tag{7}$$

which can be expressed with respect to (w.r.t.) $\mathbf{g}$ as $\mathbf{g}^H\mathbf{C}\mathbf{g} \leq P_R$ where[5]

$$\mathbf{C} = \sigma_{\mathrm{R}}^2\mathbf{I}_{M_{\mathrm{R}}^2} + \sum_{l=1}^{L}\sum_{k=1}^{2} p_{k,l}((\mathbf{h}_{k,l}\mathbf{h}_{k,l}^{\mathrm{H}})^{\mathrm{T}} \otimes \mathbf{I}_{M_{\mathrm{R}}}). \tag{8}$$

The aim is to design the AF amplification matrix $\mathbf{G}$ in order to maximize the sum-rate R_{sum}. Therefore, the design problem (i.e., sum-rate maximization) in MIMO AF relay networks with L operators can be cast as the following problem:

$$\begin{aligned} \max_{\mathbf{g}} \quad & \frac{1}{2}\sum_{l=1}^{L}\sum_{k=1}^{2}\log_2\left(1+\frac{\mathbf{g}^{\mathrm{H}}\mathbf{\Phi}_{k,l}\mathbf{g}}{\mathbf{g}^{\mathrm{H}}(\mathbf{\Upsilon}_{k,l}+\mathbf{\Delta}_{k,l})\mathbf{g}+\sigma_{k,l}^2}\right) \\ \text{s. t.} \quad & \mathbf{g}^H\mathbf{C}\mathbf{g} \leq P_R. \end{aligned} \tag{9}$$

Note that the inequality constraint in the above problem is active (i.e. satisfied with equality) at the optimal point. More precisely, assume that $\mathbf{g}$ is an optimal solution to (9) with $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_0 < P_R$. Then a scaled version of $\mathbf{g}$ which satisfies the constraint with equality, i.e. $\mathbf{g}_1 = \sqrt{P_R/P_0}\,\mathbf{g}$, will lead to a larger objective value. This contradicts the optimality assumption of $\mathbf{g}$. Considering this observation, the optimization in (9) can be equivalently recast as the following problem[6]:

$$\begin{aligned} \max_{\mathbf{g}} \quad & \prod_{l=1}^{L}\prod_{k=1}^{2}\frac{\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}}{\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}} \\ \text{s. t.} \quad & \mathbf{g}^H\mathbf{C}\mathbf{g} = P_R \end{aligned} \tag{10}$$

where we have used the following definitions:

$$\begin{aligned} \mathbf{B}_{k,l} &= \mathbf{\Upsilon}_{k,l} + \mathbf{\Delta}_{k,l} + \frac{\sigma_{k,l}^2}{P_R}\mathbf{C}, \\ \mathbf{A}_{k,l} &= \mathbf{B}_{k,l} + \mathbf{\Phi}_{k,l}. \end{aligned} \tag{11}$$

The objective function of the problem (10) consists of the product of several fractional quadratic functions. This problem is non-convex and belongs to a class of NP-hard problems in general [1].

[5]The derivations of (7) and (8) are similar to those developed in Appendix A.

[6]In (10) we let k run from 1 to 2 as in (9); however, from a mathematical point of view, the suggested approach can handle an arbitrary interval for k.

III. SUM-RATE MAXIMIZATION

A. The Proposed Algorithm

Note that the objective function in (10) is invariant with respect to scaling; therefore, we can deal with the unconstrained problem and then scale the solution $\mathbf{g}$ such that it satisfies the constraint $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$. In this paper, we use the minorization-maximization (or majorization-minimization) technique to tackle the non-convex design problem in (10). Minorization-maximization (MaMi)[7] is an iterative technique that can be used for obtaining a solution to the general maximization problem [23] [29]:

$$\begin{aligned} \max_{\mathbf{z}} \quad & \widetilde{f}(\mathbf{z}) \\ \text{s. t.} \quad & c(\mathbf{z}) \leq 0. \end{aligned} \tag{12}$$

Each iteration (say the κ^{th} iteration) of MaMi consists of two steps (see Fig. 2):

- Minorization Step: Finding $\widetilde{p}^{(\kappa)}(\mathbf{z})$ such that its maximization is simpler than that of $\widetilde{f}(\mathbf{z})$ and $\widetilde{p}^{(\kappa)}(\mathbf{z})$ minorizes $\widetilde{f}(\mathbf{z})$, i.e.,

$$\begin{aligned} \widetilde{p}^{(\kappa)}(\mathbf{z}) &\leq \widetilde{f}(\mathbf{z}), \ \ \forall \mathbf{z} \\ \widetilde{p}^{(\kappa)}\left(\mathbf{z}^{(\kappa-1)}\right) &= \widetilde{f}\left(\mathbf{z}^{(\kappa-1)}\right) \end{aligned} \tag{13}$$

  with $\mathbf{z}^{(\kappa-1)}$ being the value of $\mathbf{z}$ at the $(\kappa-1)^{th}$ iteration.

- Maximization Step: Solving the optimization problem,

$$\begin{aligned} \max_{\mathbf{z}} \quad & \widetilde{p}^{(\kappa)}(\mathbf{z}) \\ \text{s. t.} \quad & c(\mathbf{z}) \leq 0 \end{aligned} \tag{14}$$

  to obtain $\mathbf{z}^{(\kappa)}$.

Now, consider the following equivalent form of the problem in (10):

$$\max_{\mathbf{g}} \quad \sum_{l=1}^{L}\sum_{k=1}^{2}\Big[\log\left(\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}\right) - \log\left(\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}\right)\Big] \tag{15}$$

Note that the following inequality holds due to the concavity of $\log(x)$ for all $x, x_0 \in \mathbb{R}^+$:

$$\log(x) \leq \log(x_0) + \frac{1}{x_0}(x - x_0). \tag{16}$$

[7]This technique is also known as the MM algorithm in the literature [29].

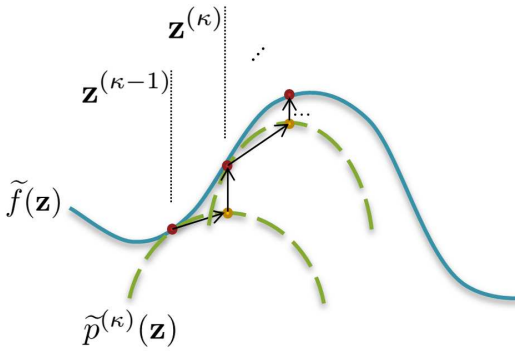


Fig. 2. An illustration of the MaMi technique.

Therefore, the term $-\log(\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g})$ can be minorized using the above inequality at any given $\mathbf{g}_0$. More precisely, setting $x_0 = \mathbf{g}_0^H\mathbf{B}_{k,l}\mathbf{g}_0$ and $x = \mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}$ leads to the following minorizer for $-\log(\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g})$:

$$\begin{aligned} &-\log(\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}) \geq \\ &-\log(\mathbf{g}_0^H\mathbf{B}_{k,l}\mathbf{g}_0) - \frac{1}{\mathbf{g}_0^H\mathbf{B}_{k,l}\mathbf{g}_0}(\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g} - \mathbf{g}_0^H\mathbf{B}_{k,l}\mathbf{g}_0). \end{aligned} \tag{17}$$

Additionally, substituting the term $-\log(\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g})$ in (15) with the above minorizer (and neglecting the constants) leads to the following maximization problem at the $(\kappa+1)^{th}$ iteration:

$$\max_{\mathbf{g}} \quad \sum_{l=1}^{L}\sum_{k=1}^{2}\left[\log(\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}) - \frac{1}{(\mathbf{g}^{(\kappa)})^H\mathbf{B}_{k,l}\mathbf{g}^{(\kappa)}}\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}\right]. \tag{18}$$

Inspired by the rich literature on semidefinite relaxation, we note that by considering $\mathbf{X} = \mathbf{g}\mathbf{g}^H$ as the optimization variable in (18) and dropping the rank-1 constraint, a convex alternative of (18) can be obtained at each iteration (see e.g., [30]). Once a solution $\mathbf{X}$ is obtained, the optimized vector $\mathbf{g}$ should be then synthesized from $\mathbf{X}$. However, there is no guarantee for a rank-1 solution[8] $\mathbf{X}$, and hence, this approach is associated with a synthesis loss [31]. In addition, applying the relaxation leads to iteratively solving a MAXDET problem possessing a high computational burden (a similar algorithm has been suggested in [1]). Instead, in the sequel, we devise a computationally efficient method that increases the objective value at each iteration and guarantees the first-order optimality condition for the solution $\mathbf{g}$ (under some mild

[8]Note that by employing the aforementioned semidefinite relaxation, a rank-1 solution $\mathbf{X}$ can be obtained just for the single operator case with $L = 1$ (see [1] for details).

conditions, see [23], [30] for details). To this end, we proceed by finding a minorizer for the term $\log(\mathrm{tr}\{\mathbf{A}_{k,l}\mathbf{X}\})$ as a function of $\mathbf{g}$ using the following Lemma.

Lemma 1. *Let* $s(\mathbf{x}) = -\log(\mathbf{x}^H\mathbf{T}\mathbf{x})$ *and* $\mathbf{x}^H\mathbf{C}\mathbf{x} = P$ *for any positive-definite matrices* $\mathbf{T}, \mathbf{C}$ *in* $\mathbb{C}^{N\times N}$ *as well as* $P \in \mathbb{R}^+$*. Then, the following inequality holds* $\forall\ \mathbf{x}, \mathbf{x}_0$*:*

$$s(\mathbf{x}) \leq s(\mathbf{x}_0) + \Re\left(\mathbf{b}^H(\mathbf{x}-\mathbf{x}_0)\right) + (\mathbf{x}-\mathbf{x}_0)^H\mathbf{U}(\mathbf{x}-\mathbf{x}_0) \tag{19}$$

where

$$\begin{aligned} \mathbf{b} &= \left(\frac{-2}{\mathbf{x}_0^H\mathbf{T}\mathbf{x}_0}\right)\mathbf{T}\mathbf{x}_0 \\ \mathbf{U} &= \left(\frac{4P}{\mathbf{w}_1^H\mathbf{C}\mathbf{w}_1} + \epsilon\right)\mathbf{I} \end{aligned} \tag{20}$$

and where $\mathbf{w}_1$ *is the principal eigenvector of* $\mathbf{T}$ *and* $\epsilon > 0$ *is arbitrary.*

Proof: See Appendix B ■

Assume that $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$ at each iteration (see the Remark 1 below). Now observe that using the above lemma, the following minorizer is obtained for the term $\log(\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g})$ at any given $\mathbf{g}_0$:

$$\begin{aligned} \log(\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}) \ \geq\ & \log(\mathbf{g}_0^H\mathbf{A}_{k,l}\mathbf{g}_0) - \Re\left((\mathbf{b}_{k,l})^H(\mathbf{g}-\mathbf{g}_0)\right) \\ & -(\mathbf{g}-\mathbf{g}_0)^H\mathbf{U}_{k,l}(\mathbf{g}-\mathbf{g}_0) \end{aligned} \tag{21}$$

where $\mathbf{b}_{k,l}$ and $\mathbf{U}_{k,l}$ are related to $\mathbf{A}_{k,l}$ and can be calculated by employing (20) in the Lemma (see (23) below). Based on (18) and (21), we consider the minimization of the following criterion w.r.t. $\mathbf{g}$ at the $(\kappa+1)^{th}$ iteration of the proposed method:

$$\begin{aligned} \sum_{l=1}^{L}\sum_{k=1}^{2}\Bigg[& \mathbf{g}^H\left(\frac{\mathbf{B}_{k,l}}{(\mathbf{g}^{(\kappa)})^H\mathbf{B}_{k,l}\mathbf{g}^{(\kappa)}} + \mathbf{U}_{k,l}\right)\mathbf{g} \\ & + \Re\left(\left(\mathbf{b}_{k,l} - 2\mathbf{U}_{k,l}\mathbf{g}^{(\kappa)}\right)^H\mathbf{g}\right)\Bigg] \end{aligned} \tag{22}$$

with

$$\begin{aligned} \mathbf{b}_{k,l} &= \left(\frac{-2}{(\mathbf{g}^{(\kappa)})^H\mathbf{A}_{k,l}\mathbf{g}^{(\kappa)}}\right)\mathbf{A}_{k,l}\mathbf{g}^{(\kappa)}, \\ \mathbf{U}_{k,l} &= \left(\frac{4P_R}{\widetilde{\mathbf{w}}_{k,l}^H\mathbf{C}\widetilde{\mathbf{w}}_{k,l}} + \epsilon\right)\mathbf{I}, \end{aligned} \tag{23}$$

and $\widetilde{\mathbf{w}}_{k,l}$ denoting the principal eigenvector of $\mathbf{A}_{k,l}$. The above optimization problem can be recast as the following unconstrained QP:

$$\min_{\mathbf{g}} \quad \mathbf{g}^H \mathbf{Q}^{(\kappa)} \mathbf{g} + \Re\left(\left(\mathbf{q}^{(\kappa)}\right)^H \mathbf{g}\right) \tag{24}$$

where

$$\mathbf{Q}^{(\kappa)} = \sum_{l=1}^{L}\sum_{k=1}^{2}\left[\frac{\mathbf{B}_{k,l}}{\left(\mathbf{g}^{(\kappa)}\right)^H \mathbf{B}_{k,l}\mathbf{g}^{(\kappa)}} + \mathbf{U}_{k,l}\right], \tag{25}$$

$$\mathbf{q}^{(\kappa)} = \sum_{l=1}^{L}\sum_{k=1}^{2}\left[\mathbf{b}_{k,l} - 2\mathbf{U}_{k,l}\mathbf{g}^{(\kappa)}\right]. \tag{26}$$

Note that $\mathbf{B}_{k,l} \succeq \mathbf{0}$, and also $\mathbf{U}_{k,l} \succ \mathbf{0}$ because it is a scaled version of identity matrix $\mathbf{I}$ with a positive scalar. Therefore, the matrix $\mathbf{Q}^{(\kappa)}$ is positive-definite at each iteration. Consequently, the problem in (24) is strictly convex w.r.t. $\mathbf{g}$. The unique solution to this optimization is obtained by solving the system of linear equations $2\mathbf{Q}^{(\kappa)}\mathbf{g} + \mathbf{q}^{(\kappa)} = \mathbf{0}$, viz.

$$\mathbf{g} = -\frac{1}{2}\left(\mathbf{Q}^{(\kappa+1)}\right)^{-1}\mathbf{q}^{(\kappa)}. \tag{27}$$

It is worth noting that the solution $\mathbf{g}$ to the above system of linear equations can also be obtained via directly solving the linear system using more efficient techniques, and thus avoiding the inverse (see e.g., [32], [33], and the references therein).

Remark 1: Note that the above solution $\mathbf{g}$ does not necessarily satisfy the constraint $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$ of the original problem (10) at each iteration. As mentioned before, we can scale the obtained solution at the convergence to deal with this issue as the objective function in (10) is scale invariant. However, the derivation of the matrix $\mathbf{U}_{k,l}$ in Lemma 1 requires the satisfaction of the constraint at each iteration; see Appendix B for details. Therefore, we need to scale the obtained $\mathbf{g}$ at each iteration such that $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$. Note also that the scaling does not affect the convergence of the sequence of the objective function values associated with the problem (10). ■

Table II summarizes the steps of the proposed method for relay beamformer design to maximize the communication sum-rate. The suggested method improves the value of the objective function at each iteration (see Section IV-B). Note that the computational complexity of the method is linear with the number of iterations $\bar{N}$. Furthermore, each iteration of the algorithm consists of solving a strictly convex problem (24) using either the closed-form solution in

TABLE II

THE PROPOSED METHOD FOR SUM-RATE MAXIMIZATION VIA RELAY BEAMFORMER DESIGN

Step 0: Initialize $\mathbf{g}$ with a random vector in $\mathbb{C}^{M_R^2}$ (and scale it such that $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$); set $\kappa = 0$.
Step 1: Compute $\mathbf{Q}^{(\kappa)}$ and $\mathbf{q}^{(\kappa)}$ using (25).
Step 2: Solve the convex problem in (24) using either the closed-from expression (27) or direct methods (for solving the associated system of linear equations) to obtain $\mathbf{g}^{(\kappa+1)}$.
Step 3: Scale the obtained solution $\mathbf{g}^{(\kappa+1)}$ such that $(\mathbf{g}^{(\kappa+1)})^H\mathbf{C}\mathbf{g}^{(\kappa+1)} = P_R$; set $\kappa \leftarrow \kappa + 1$.
Step 5: Repeat steps 1-3 until a pre-defined stop criterion is satisfied, e.g. $\lvert f^{(\kappa+1)} - f^{(\kappa)}\rvert \leq \xi$ (where f denotes the objective function of the problem (10)) for some $\xi > 0$.

(27) or direct/iterative methods for solving systems of linear equations [32], [33]. The steps for computing the solution can be implemented e.g. via the algorithms in [24] (for matrix multiplications) with $\mathcal{O}(n^{2.3})$ complexity where $n = M_R^2$ is the problem dimension. The devised method can handle problems of dimension on the order of 10^3 variables on an ordinary PC within a few minutes. The computational efficiency of this method makes it potentially useful in medium-scale MIMO systems (see [34] for descriptions of a recently developed prototype of such a system). We herein also remark on the fact that the MaMi algorithms were originally developed to achieve a very low computational burden by avoiding complicated matrix inversions that are an indispensable part of off-the-shelf optimization packages; see, e.g. [35]. Also, comparisons with various methods show that MaMi algorithms are usually difficult to beat in terms of stability and computational simplicity [29].

B. Weighted Sum-Rate Maximization

In practice, the users of specific operators may have a higher priority compared to others. In such cases, a maximization of the weighted sum-rate becomes of interest. The weighted sum-rate is given by

$$\frac{1}{2}\sum_{l=1}^{L}\sum_{k=1}^{2} w_{k,l}\log_2(1+\eta_{k,l}) \tag{28}$$

with $w_{k,l}$ being the (non-negative) weights associated with the k^{th} user of the l^{th} operator. Similar to the sum-rate maximization case, the corresponding optimization problem can be cast as

$$\begin{aligned} \max_{\mathbf{g}} \quad & \prod_{l=1}^{L}\prod_{k=1}^{2}\left(\frac{\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}}{\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}}\right)^{w_{k,l}} \\ \text{s. t.} \quad & \mathbf{g}^H\mathbf{C}\mathbf{g} = P_R. \end{aligned} \tag{29}$$

The above problem can be dealt with via the proposed method in this paper after some minor modifications. To see this, consider the following equivalent form for the problem in (29):

$$\max_{\mathbf{g}} \quad \sum_{l=1}^{L}\sum_{k=1}^{2} w_{k,l}\Big[\log\left(\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}\right) - \log\left(\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}\right)\Big]$$

and note that solutions to the above problem, which is a modified version of (15), can be obtained via the algorithm in Table II, using

$$\mathbf{Q}^{(\kappa)} = \sum_{l=1}^{L}\sum_{k=1}^{2} w_{k,l}\left[\frac{\mathbf{B}_{k,l}}{\left(\mathbf{g}^{(\kappa)}\right)^H\mathbf{B}_{k,l}\mathbf{g}^{(\kappa)}} + \mathbf{U}_{k,l}\right], \tag{30}$$

$$\mathbf{q}^{(\kappa)} = \sum_{l=1}^{L}\sum_{k=1}^{2} w_{k,l}\left[\mathbf{b}_{k,l} - 2\mathbf{U}_{k,l}\mathbf{g}^{(\kappa)}\right], \tag{31}$$

in lieu of (25)-(26).

IV. Sum-rate upper bound and some computational aspects

A. Sum-Rate Upper Bound

In the following, we derive an upper bound on the objective function of (10), and the associated sum-rate metric. Note that the boundedness of (10) is a key fact for the convergence of the proposed method—see Section IV-B below for details. We observe that each term of the product on the right-hand side of

$$2^{2R_{sum}} = \prod_{l=1}^{L}\prod_{k=1}^{2}\frac{\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}}{\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}} \tag{32}$$

can be bounded from above by considering the related generalized eigenvalue problem, viz.

$$\frac{\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}}{\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}} \leq \lambda_{max}\left\{\mathbf{B}_{k,l}^{-1}\mathbf{A}_{k,l}\right\}. \tag{33}$$

Moreover, as the sum-rate is invariant with respect to permutations of the matrices $\{\mathbf{A}_{k,l}\}$ and $\{\mathbf{B}_{k,l}\}$ within the products, the upper bound may be strengthened by considering such

permutations. More precisely, let $\pi(k,l) : \{1,2\} \times \{1,\cdots,L\} \rightarrow \{1,2\} \times \{1,\cdots,L\}$ denote a generic permutation function over all possible (k,l). Then, according to the generalized eigenvalue upper bound,

$$\frac{\mathbf{g}^H \mathbf{A}_{\pi(k,l)} \mathbf{g}}{\mathbf{g}^H \mathbf{B}_{k,l} \mathbf{g}} \leq \lambda_{max} \left\{ \mathbf{B}_{k,l}^{-1} \mathbf{A}_{\pi(k,l)} \right\} \tag{34}$$

which implies

$$R_{sum} \leq \frac{1}{2} \log_2 \left(\min_{\pi(k,l)} \left\{ \prod_{l=1}^{L} \prod_{k=1}^{2} \lambda_{max} \left\{ \mathbf{B}_{k,l}^{-1} \mathbf{A}_{\pi(k,l)} \right\} \right\} \right). \tag{35}$$

Next note that the equality in (34) is attained when $\mathbf{g}$ is a principal eigenvector of the matrix $\mathbf{B}_{k,l}^{-1} \mathbf{A}_{\pi(k,l)}$. As a consequence, the upper bound in (35) is attained when matrices $\mathbf{A}$, $\mathbf{B}$, and $\{\mathbf{U}_{k,l}\}$ (with $\mathbf{B}$, and $\{\mathbf{U}_{k,l}\}$ being invertible) exist such that $\mathbf{A}_{\pi(k,l)} = \mathbf{A}\mathbf{U}_{k,l}$ and $\mathbf{B}_{k,l} = \mathbf{B}\mathbf{U}_{k,l}$, for all $(k,l) \in \{1,2\} \times \{1,\cdots,L\}$. This shows that the bound in (35) is tight and it can not be improved upon unless the class of matrices $\{\mathbf{A}_{k,l}, \mathbf{B}_{k,l}\}$ is restrained.

B. Convergence

In order to study the convergence of the devised approach, observe that

$$\begin{aligned} f\left(\mathbf{g}^{(\kappa-1)}\right) &= p^{(\kappa)}\left(\mathbf{g}^{(\kappa-1)}\right) \\ &\leq p^{(\kappa)}\left(\mathbf{g}^{(\kappa)}\right) \leq f\left(\mathbf{g}^{(\kappa)}\right) \end{aligned} \tag{36}$$

where $p^{(\kappa)}(.)$ is the minorizer associated with the objective $f(.)$ at the κ^{th} iteration. The first inequality in (36) holds due to the maximization step at the κ^{th} iteration, whereas the second inequality comes from the definition of the minorizer (see (13)). This monotonically increasing property together with the derived upper bound in (35) guarantees the convergence of the sequence of the objective values $\{f\left(\mathbf{g}^{(\kappa)}\right)\}$, and hence of the sum-rate metric. Note that the obtained solutions via the proposed method are stationary points of the problem (under some mild conditions [23]) satisfying the first-order optimality criterion for the non-convex problem (10).

C. Practical Real-Time Applications

The communication channels $\{\mathbf{h}_{k,l}\}$ are subject to change with time in real-world applications in particular due to the relative motions of users/relay/scatterers. The time intervals for which

the channels can be assumed to be invariant depend on the Doppler spreads of the channels (see e.g., [36], [37] for details). Note that for any new set of channel parameters $\{\mathbf{h}_{k,l}\}$, a new beamforming matrix $\mathbf{G}$ is to be designed. Therefore, the convergence speed (computational complexity) associated with the design method plays an important role in the applicability of the method. In addition to general results on the computational efficiency of the proposed method, an interesting aspect of the proposed method is that the quality of the obtained solution and the convergence speed depend on the employed starting point (see Section V and [23] for details). Therefore, in real-time applications where the communication channels $\{\mathbf{h}_{k,l}\}$ change with time, the proposed method can quickly converge if initialized with the preceding solution $\mathbf{g}$. Note also that in practical real-time applications, the method can be implemented more efficiently (e.g., via implementation in C language, parallelization, and so on) and be run on powerful digital signal processors (DSP/FPGA) to speed up the convergence significantly (see the discussion of Table II as well).

V. Numerical Examples and Discussions

In this section, the performance of the proposed method is evaluated via Monte-Carlo simulations. An AF based bidirectional MIMO relay network with L operators and M_R antennas at the relay is considered. The variances of the Gaussian noises for the relay and users are assumed to be equal, i.e., $\sigma_R^2 = \sigma_{k,l}^2 = \sigma_n^2$. For the sake of comparison, we use the same power allocation as considered in other related works (see e.g., [1], [6] and the references therein); namely, we assume that the transmit powers of the relay and users are identical, i.e., $P_R = p_{k,l} = p$. The signal-to-noise ratio (SNR) is defined as p/σ_n^2. Moreover, the normalized distance between k^{th} user of the l^{th} operator and the relay is represented by $d_{k,l}$. For simplicity and without loss of generality, we assume that $d_{1,l} = d_1$ and $d_{2,l} = d_2$ (with $d_1 + d_2 = 1$). Therefore, the near-far (N/F) ratio is defined as d_1/d_2. The Rayleigh flat fading channel vectors $\{\mathbf{h}_{k,l}\}$ are reciprocal and spatially uncorrelated. The path loss exponent is assumed to be 3 in all simulations, thus the fading variances are proportional to $1/d_{k,l}^3$ [36], [37]. As a result, $\{\mathbf{h}_{k,l}\}$ are modeled as independent Gaussian random vectors with $\mathbf{h}_{k,l} \sim \mathcal{CN}\left(\mathbf{0}, (d_0/d_{k,l})^3 \mathbf{I}\right)$ where $d_0 = 0.1$ is the considered reference point. All the results are presented considering 100 realizations of the associated fading channels. As to the convergence of the proposed method, we consider $\xi = 10^{-3}$ in Table II. The QP of the step 1 of the proposed method (see Table II) is solved using the

embedded MATLAB function for directly solving systems of linear equations[9].

We begin by investigating the effect of the SNR on the sum-rate in a symmetric scenario (i.e., $d_1 = d_2$). The sum-rate values associated with the proposed method as well as the POTDC[10] method of [1] versus SNR are shown in Fig. 3 for $M_R = 4$ and $M_R = 8$ with $L = 2$. As expected, the sum-rate is increasing with respect to SNR. Furthermore, the results of the proposed method are slightly better than those of the method in [1] because the proposed method circumvents the synthesis loss associated with POTDC. This figure also includes the results for ZF and MRC methods with $M_R \in \{4, 8\}$ and $M_R = 20$ (that can be considered as a medium-scale MIMO scenario). It can be observed that the proposed method outperforms well ZF and MRC methods. This observation is compatible with the fact that ZF and MRC are merely nearly optimal when the number of antennas diverges to infinity. Next, we study the effect of the N/F ratio. Fig. 4 plots the sum-rate values versus different N/F ratios ($L = 2$, SNR=20dB). The proposed method achieves better results in the whole interval of the N/F ratio when compared to other methods. Note that the N/F ratio is defined as d_1/d_2 and the maximum rate is achieved in the symmetric scenario where the relay is in the middle of users (see [4] for more details and [1], [6] for similar behaviors). Note that in the above figures, we do not include the results of POTDC method for $M_R = 20$ due to prohibitive computational burden.

It can be observed from Fig. 3 and Fig. 4 that the larger the number of antennas M_R, the larger the sum-rate. This aspect is further explored in Fig. 5 (a) and (b) where the values of sum-rate are plotted versus M_R for $L = 1$ and $L = 2$, respectively. Fig. 5 (a) also includes two ad-hoc algorithms, namely 1-D RAGES and 2-D RAGES [1] along with the upper bound [16] on the sum-rate values for $L = 1$. The monotonically increasing behavior of the sum-rate with respect to M_R is evident from these figures. This behavior can be justified by considering the fact that larger values of M_R (i.e., more antennas at the relay) provide more degrees of freedom for the design problem. For the case of $L = 1$, the differences between the sum-rate values of various methods are minor. Note that the curves associated with POTDC, 1-D RAGES, and 2-D RAGES are truncated for larger values of M_R due to the prohibitive computational burden of

[9]The reader may refer to the MATLAB command "$\mathbf{A}\backslash\mathbf{b}$" for obtaining the solution to the linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$.

[10]It has been shown that POTDC outperforms several existing methods for sum-rate maximization in the conventional MIMO scenarios (see e.g., [1], [11], [28]).

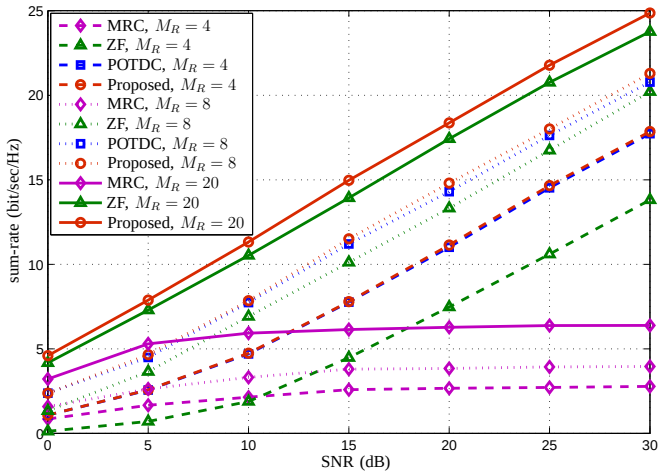


Fig. 3. The values of sum-rate associated with the proposed method and the method of [1] (i.e., POTDC), ZF, as well as MRC versus SNR for $L = 2$.

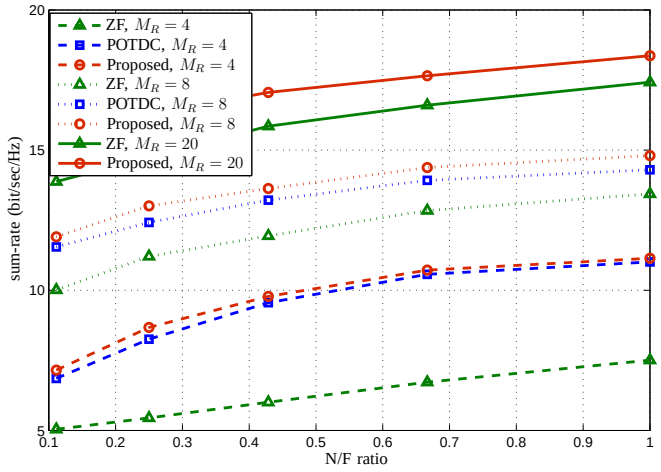
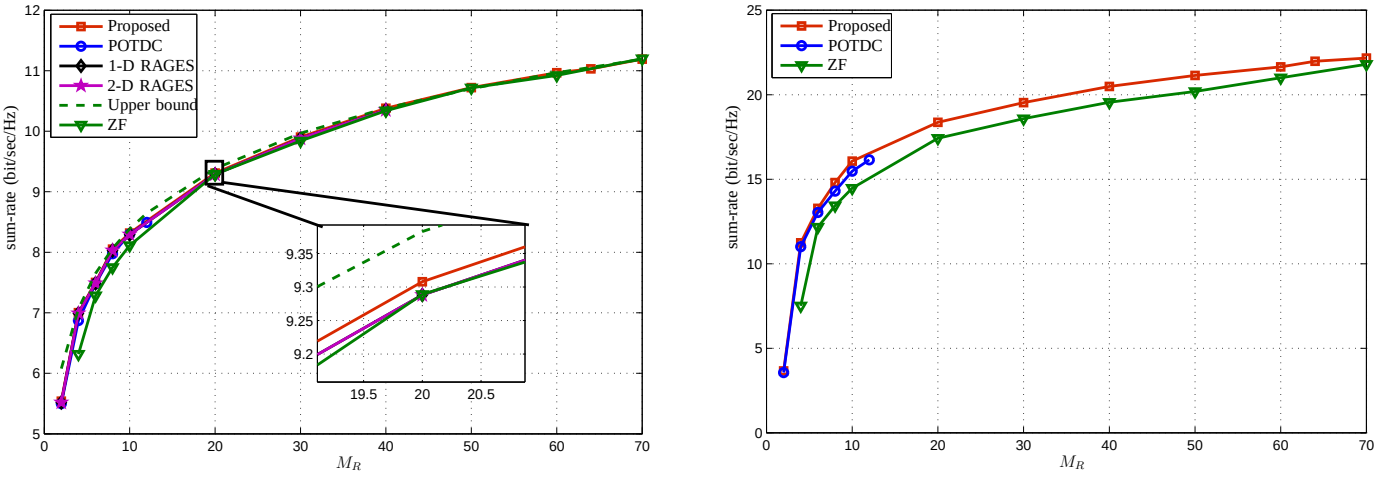


Fig. 4. The sum-rate values for various methods versus N/F ratio for $L = 2$ and SNR=20dB.

these methods. We further note that the proposed method tracks the upper bound for various M_R well.

Fig. 5 (a) and (b) plot the sum-rate values for the ZF as well[11]. It can be observed from Fig. 5(b) that for a wide range of the considered M_R, the proposed method outperforms well the ZF,

[11]We do not include the results of MRC because for $L = 1$ is very similar to ZF and for $L = 2$ is not competitive.



(a) (b)

Fig. 5. The sum-rate values for various methods versus the number of antennas M_R: (a) the case of $L = 1$, (b) the case of $L = 2$. The SNR is set to 20dB.

as expected (see the explanations related to lower-regime massive in Section I). Then, by further increasing the number of antennas, ZF tends to the obtained values by the proposed method (massive regime). Note that the borderline between the lower-regime massive and massive is not sharp and also depends on other parameters like the number of operators L; e.g., in Fig. 5(a), at smaller values of M_R we have similar sum-rate values for the proposed method and ZF (this point will be analyzed shortly–see Fig. 6 below).

It can also be observed from Fig. 5 (a) and (b) that the sum-rate for $L = 1, M_R = 2$ is larger than that of the case with $L = 2, M_R = 2$. This can be explained considering the fact that in the case of $L = 2$, the interference power for the users of the first (second) operator grows due to the existence of the interferences corresponding to users of the second (first) operator; whereas, when the system has one operator, the interference power only comes from the relay/receivers' noise. This leads to a lower sum-rate value for the system with two operators (and $M_R = 2$). Note that by increasing the number of antennas M_R, a judicious design of the relay beamformer matrix $\mathbf{G}$ decreases the interference power and as a result, larger sum-rate values will be obtained (see Fig. 5 (a) and (b)). However, for small M_R, there are not sufficient degrees of freedom (in the design problem) to circumvent the interference power associated with the second operator. Therefore, one can conclude that a large number of antennas for the relay becomes quite useful

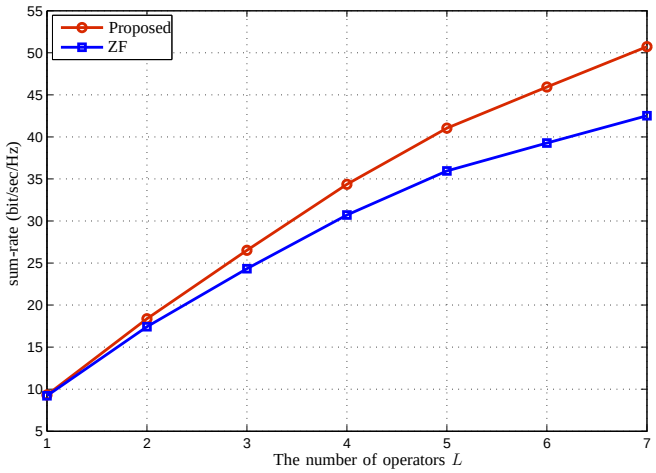


Fig. 6. The values of sum-rate associated with the proposed method and ZF versus the number of operators L. The values of M_R and SNR are set to 20 and 20dB, respectively.

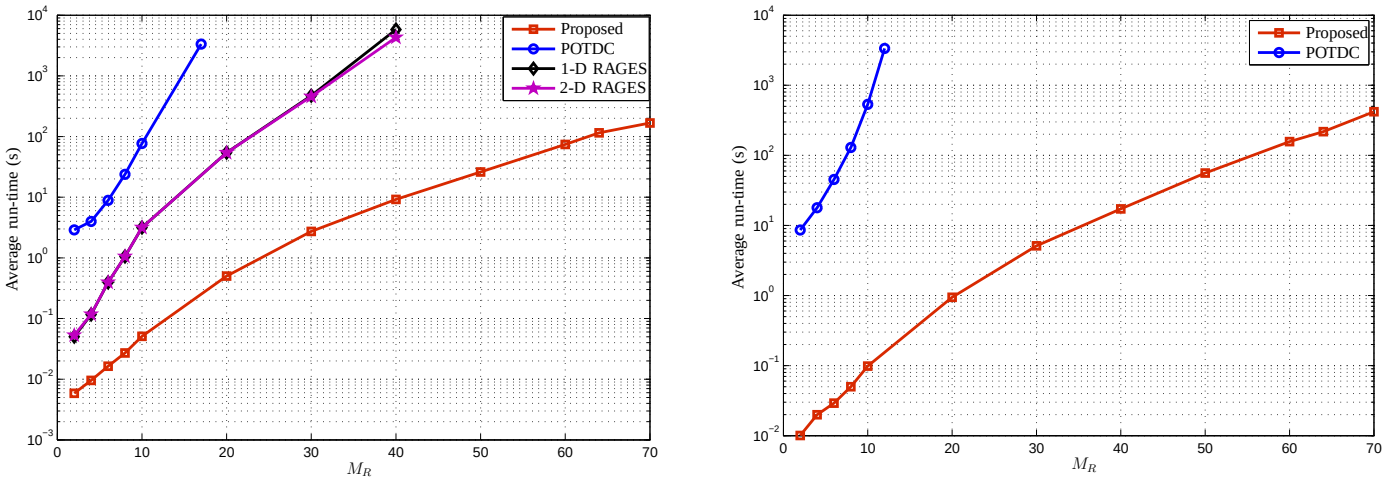


Fig. 7. The average run-time (seconds) for various methods versus the number of antennas M_R associated with Fig. 5: (a) the case of $L = 1$, and (b) the case of $L = 2$. Note that the problem dimension is given by $n = M_R^2$.

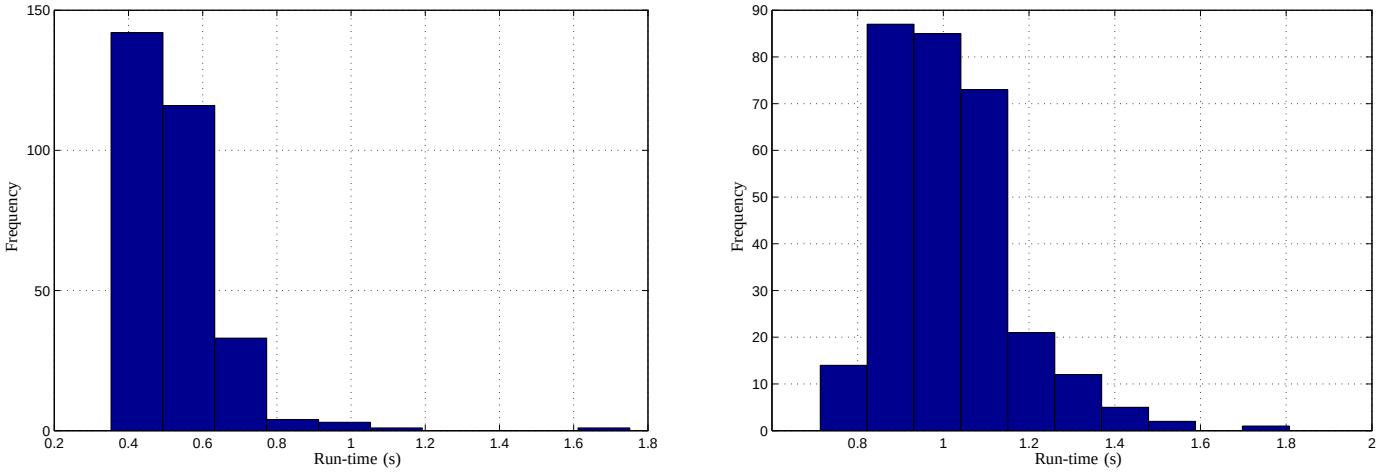
when several operators are supposed to work simultaneously.

The effect of the number of operators L is considered in Fig. 6. The figure shows the sum-rate values versus L for $M_R = 20$ and SNR=20dB. The figure includes the results of the proposed method and ZF (we do not include MRC herein as it is not competitive). Also, the POTDC has

prohibitive computational time for $L > 2$ and RAGES methods can not be applied for $L > 1$. It is observed that larger L leads to more significant gap between the proposed method and ZF. This behavior can be explained considering the fact that the asymptotical near optimality of ZF depends on the number of operators L in addition to the number of antennas M_R (see Section I and [2]).

The computational times of the various methods for tackling the sum-rate optimization problem (10) are analyzed in Fig. 7 (a) and (b), respectively. The figures illustrate the average computational time considering 10 runs of the methods with random initializations on an ordinary PC (with 8GB RAM and CPU CoRe i5). It can be seen that the POTDC method has the highest computational burden as compared to the other methods (note that the values for POTDC correspond to 10 iterations). The 1-D RAGES and 2-D RAGES algorithms can be employed for up to $M_R = 40$ [1]. It is observed that the proposed method has much lower computational cost when compared to the existing methods. To be more precise, the computational time of our method is on the order of (at most) a few seconds at $M_R \sim 40, 50$. We remark on the fact that the ZF and MRC methods have lower computational burden when compared to the proposed method; however, the resulting sum-rate values corresponding to these methods are considerably lower than that of the proposed method in conventional and medium-scale (lower-regime massive) MIMO systems (see e.g., Figs. 3, 4, 5, and 6 and the discussion in Section I). Therefore, meaningful improvements in the sum-rate values (i.e., locally optimal values) can be achieved by the proposed method in such systems at the cost of a higher computational burden as compared to the ZF/MRC (see Section IV-C for a discussion of the computational time and practical applications).

We next investigate the initialization and convergence speed/time of the proposed method. To analyze the random initialization, we report the histogram of the convergence times on a standard PC (see above) for various random initiations. We consider 300 independent random Gaussian vectors in $\mathbb{C}^{M_R^2}$ with i.i.d. elements. Fig. 8 (a) and (b) plot the aforementioned histograms for the case of $L = 1$ as well as $L = 2$, respectively (assuming $M_R = 20$ and SNR=20dB). It can be observed that the histograms are concentrated well around the corresponding averaged values in Fig. 7. The convergence time of the proposed method when it is initialized by the sub-optimal solutions is shown in Fig. 9 versus M_R. In this figure, the ZF and MRC are employed to initialize the proposed method for SNR=20dB. As expected, such starting points speed up the convergence



(a) (b)

Fig. 8. The histograms of the computational time (sec.) for the proposed method on a standard PC employing 300 random Gaussian starting points, a) $L = 1$, b) $L = 2$. The values of M_R and SNR are set to 20 and 20dB, respectively.

of the algorithm as compared to the random initialization.

VI. Conclusion

The problem of sum-rate maximization in MIMO AF relay networks with multi-operator was considered. The aim was to optimally design the relay beamforming matrix in order to maximize the communication sum-rate. The design problem was cast as the maximization of a product of many fractional QPs subject to the relay power constraint, which belongs to a class of NP-hard problems in general. We devised an iterative method based on the minorization-maximization (MaMi) technique to deal with the problem. The minorizers for the objective function terms were derived by using linear and quadratic minorizers for matrix/vector functions. The proposed method provides quality solutions to the design problem (i.e., stationary points of the problem, under some mild conditions) for an arbitrary number of operators L. Each iteration of the proposed method was dealt with via solving an unconstrained (strictly) convex QP either using a closed-form solution or by solving a system of linear equations. Numerical examples confirmed the effectiveness of the proposed method when compared to other methods in terms of the solution quality and the computational efficiency. In particular, the method could handle design problems of dimensions of up to several thousands variables (equivalently, a number of antennas

of up to $M_R \sim 70$) on an ordinary PC within a few minutes, which makes it potentially useful in medium-scale (or lower-regime massive) MIMO scenarios.

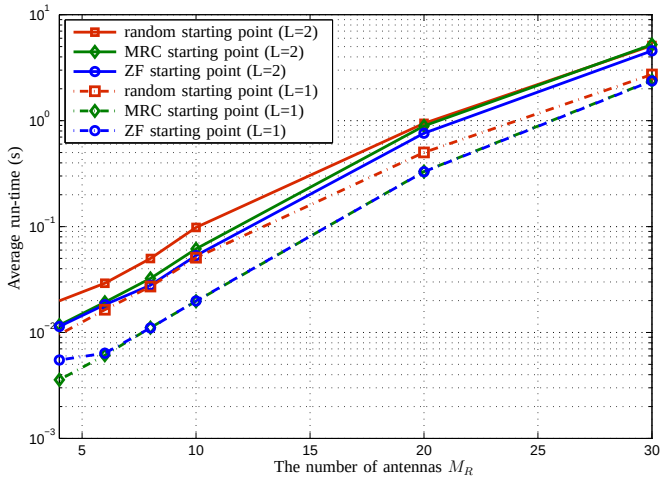


Fig. 9. The comparison of the computational times of the proposed method when initialized by suboptimal solutions and random starting point. Herein, we set SNR=20dB.

APPENDIX A
THE DERIVATION OF THE SINR EXPRESSION IN (5)

Note that the SINR $\eta_{k,l}$ for the k^{th} user of the l^{th} operator can be computed via the following expression:

$$\eta_{k,l} = \frac{\mathrm{E}\{|\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{h}_{3-k,l}x_{3-k,l}|^2\}}{\mathrm{E}\{|\sum_{\tilde{k},\tilde{l}\neq l}\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{h}_{\tilde{k},\tilde{l}}x_{\tilde{k},\tilde{l}}|^2\} + \mathrm{E}\{|\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{n}_R|^2\} + \sigma_{k,l}^2} \tag{37}$$

in which the numerator can be expanded as

$$\begin{aligned}
&\mathrm{E}\{|\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{h}_{3-k,l}x_{3-k,l}|^2\} \\
&= \mathrm{E}\{|x_{3-k,l}|^2\}(\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{h}_{3-k,l})(\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{h}_{3-k,l})^* \\
&= p_{k,l}\mathrm{tr}\{(\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{h}_{3-k,l})^H\}\mathrm{tr}\{\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{h}_{3-k,l}\} \\
&= p_{k,l}\mathrm{vec}(G)^H\left((\mathbf{h}_{3-k,l}^T\otimes\mathbf{h}_{k,l}^T)^H(\mathbf{h}_{3-k,l}^T\otimes\mathbf{h}_{k,l}^T)\right)\mathrm{vec}(G)
\end{aligned}$$

In the above we have used the fact that the scalar $\mathbf{h}_{k,l}^T\mathbf{G}\mathbf{h}_{3-k,l}$ can be alternatively written as $(\mathbf{h}_{3-k,l}^T\otimes\mathbf{h}_{k,l}^T)\mathrm{vec}(G)$ considering the Kronecker product property $\mathrm{tr}\{\mathbf{ABC}\} = (\mathbf{C}^T\otimes\mathbf{A})\mathrm{vec}(\mathbf{B})$ [38]. Now by defining $\mathbf{g} = \mathrm{vec}(\mathbf{G})$ and

$$\mathbf{\Phi}_{k,l} = p_{k,l}(\mathbf{h}_{3-k,l}^T\otimes\mathbf{h}_{k,l}^T)^H(\mathbf{h}_{3-k,l}^T\otimes\mathbf{h}_{k,l}^T)$$

the numerator of $\eta_{k,l}$ in (37) can be rewritten as $\mathbf{g}^H\mathbf{\Phi}_{k,l}\mathbf{g}$. Using similar calculations, the terms in the denominator of the SINR $\eta_{k,l}$ in (37) can be straightforwardly expressed as they are stated in (5) and (6). Note that (7) and (8) can also be verified via similar techniques.

APPENDIX B
PROOF OF LEMMA 1

We begin the proof by separating the real and imaginary parts of the variable $\mathbf{x} \in \mathbb{C}^N$ as $[\mathbf{z}^T \ \mathbf{y}^T]^T \in \mathbb{R}^{2N}$. Next, we consider the Taylor expansion of the function $s\left([\mathbf{z}^T \ \mathbf{y}^T]^T\right)$ which leads to a standard quadratic majorizer [29]. It can be verified that by employing straightforward techniques, the aforementioned majorizer can be expressed w.r.t. $\mathbf{x}$ as

$$\begin{aligned} s(\mathbf{x}) \ &\leq \ s(\mathbf{x}_0) + \Re\left(\nabla s(\mathbf{x})^H|_{\mathbf{x}=\mathbf{x}_0}(\mathbf{x}-\mathbf{x}_0)\right) \\ &\quad +(\mathbf{x}-\mathbf{x}_0)^H\mathbf{U}(\mathbf{x}-\mathbf{x}_0) \end{aligned} \tag{38}$$

for all $\mathbf{x}, \mathbf{x}_0$. Note that the existence of $\mathbf{U} \succeq \mathbf{0}$ such that $\nabla^2 s(\mathbf{x}) \preceq \mathbf{U}$ for all $\mathbf{x}$ guarantees holding of the above inequality [29]. In the sequel, we derive the matrix bound $\mathbf{U}$ on $\nabla^2 s(\mathbf{x})$. Let $h(\mathbf{x}) = \mathbf{x}^H\mathbf{T}\mathbf{x}$ and $\mathbf{x} = [\mathbf{z}^T \ \mathbf{y}^T]^T$; using the results of [38], [39] it is verified that

$$\begin{aligned} \nabla s(\mathbf{x}) \ &= \ \frac{-1}{h(\mathbf{x})}\nabla h(\mathbf{x}) = \frac{-2\mathbf{T}\mathbf{x}}{\mathbf{x}^H\mathbf{T}\mathbf{x}}, \\ \nabla^2 s(\mathbf{x}) \ &= \ \frac{-1}{h(\mathbf{x})}\nabla^2 h(\mathbf{x}) + \frac{\nabla h(\mathbf{x})\nabla h(\mathbf{x})^H}{\nabla^2 h(\mathbf{x})} \\ &= \ \frac{-2\mathbf{T}}{\mathbf{x}^H\mathbf{T}\mathbf{x}} + \frac{4\mathbf{T}\mathbf{x}\mathbf{x}^H\mathbf{T}}{(\mathbf{x}^H\mathbf{T}\mathbf{x})^2}. \end{aligned} \tag{39}$$

Note that $\mathbf{T} \succeq \mathbf{0}$, and therefore, the first term in the expression of $\nabla^2 s(\mathbf{x})$ is negative-definite. As a result, it suffices to obtain $\gamma > 0$ such that

$$\frac{4\mathbf{T}\mathbf{x}\mathbf{x}^H\mathbf{T}}{(\mathbf{x}^H\mathbf{T}\mathbf{x})^2} \preceq \gamma\mathbf{I}. \tag{40}$$

Herein, we remark on the fact that several algebraic bounds can be obtained satisfying (40); however, the tightness of the bound affects the convergence speed and quality of the solution. Therefore, in what follows, we derive the bound considering an optimization problem. Indeed, as the matrix $\mathbf{T}\mathbf{x}\mathbf{x}^H\mathbf{T}$ is rank-one, we can select γ as $\gamma > 4\zeta$ with

$$\zeta = \max_{\mathbf{x}} \ \frac{\mathbf{x}^H\mathbf{T}^2\mathbf{x}}{(\mathbf{x}^H\mathbf{T}\mathbf{x})^2} \tag{41}$$

The positive definiteness of the matrix $\mathbf{T}$ ensures existence of the full-rank (square) matrix $\mathbf{V}$ such that $\mathbf{T} = \mathbf{V}\mathbf{V}^H$. Let $\mathbf{a} = \mathbf{V}^H\mathbf{x}$ and consider the following equivalent expression for the above objective function w.r.t. $\mathbf{a}$:

$$\left(\frac{\mathbf{a}^H(\mathbf{V}^H\mathbf{V})\mathbf{a}}{\mathbf{a}^H\mathbf{a}}\right)\left(\frac{1}{\mathbf{a}^H\mathbf{a}}\right).$$

The latter change of variables leads to the following optimization problem[12]:

$$\max_{\mathbf{a}} \ \left(\frac{\mathbf{a}^H(\mathbf{V}^H\mathbf{V})\mathbf{a}}{\mathbf{a}^H\mathbf{a}}\right)\left(\frac{1}{\mathbf{a}^H\mathbf{a}}\right) \tag{42}$$

Now, we proceed by solving the above optimization problem in order to obtain ζ. Observe that the first factor in the objective of the problem in (42) is independent of $\|\mathbf{a}\|_2$. Indeed, by noting that the expression $\frac{\mathbf{a}^H(\mathbf{V}^H\mathbf{V})\mathbf{a}}{\mathbf{a}^H\mathbf{a}}$ is a Rayleigh quotient [38], one can immediately obtain its maximum value given by $\lambda_{max}(\mathbf{V}^H\mathbf{V})$. This value is achievable by choosing $\mathbf{a}$ to have the same *direction* (i.e. $\mathbf{a}/\|\mathbf{a}\|_2$) as the principal eigenvector of the matrix $\mathbf{V}^H\mathbf{V}$, denoted by $\mathbf{v}_1$. Once the direction of the optimal $\mathbf{a}$ (i.e. $\mathbf{a}/\|\mathbf{a}\|_2 = \mathbf{v}_1$) is obtained, the value of $\|\mathbf{a}\|_2$ can be calculated by considering the constraint $\mathbf{x}^H\mathbf{C}\mathbf{x} = P$. More concretely, we have

$$(\mathbf{V}^{-H}\mathbf{a})^H\mathbf{C}(\mathbf{V}^{-H}\mathbf{a}) = P$$

which yields

$$\|\mathbf{a}\|_2 = \sqrt{\frac{P}{\mathbf{v}_1^H\mathbf{V}^{-1}\mathbf{C}\mathbf{V}^{-H}\mathbf{v}_1}}. \tag{43}$$

The maximum value of the objective function in (42) is thus given by

$$\begin{aligned}\zeta &= \left(\frac{P}{\mathbf{v}_1^H\mathbf{V}^{-1}\mathbf{C}\mathbf{V}^{-H}\mathbf{v}_1}\right)\lambda_{max}(\mathbf{V}^H\mathbf{V}) \qquad (44)\\ &= \left(\frac{P}{\mathbf{v}_1^H\mathbf{V}^{-1}\mathbf{C}\mathbf{V}^{-H}\mathbf{v}_1}\right)\lambda_{max}(\mathbf{T}).\end{aligned}$$

Consequently, we can select $\gamma = 4\zeta + \epsilon$ for some $\epsilon > 0$ to bound $\nabla^2 s(\mathbf{x})$. Note that as $\mathbf{T} \succ \mathbf{0}$, the matrix $\mathbf{V}$ is invertible. Finally, using (38), (39), (44), and the discussions above we obtain

$$\begin{aligned}\mathbf{b} &\triangleq \nabla s(x)|_{\mathbf{x}=\mathbf{x}_0} = \frac{-2\mathbf{T}\mathbf{x}_0}{\mathbf{x}_0^H\mathbf{T}\mathbf{x}_0} \qquad (45)\\ \mathbf{U} &\triangleq \left(\frac{4P}{\mathbf{w}_1{}^H\mathbf{C}\mathbf{w}_1} + \epsilon\right)\mathbf{I}\end{aligned}$$

where $\mathbf{w}_1$ is the principal eigenvector of the matrix $\mathbf{T}$; and hence the proof is complete.

[12]Due to the fact that $\mathbf{T} \succ \mathbf{0}$, the optimal $\mathbf{x}$ can be uniquely determined via the optimal $\mathbf{a}$.

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