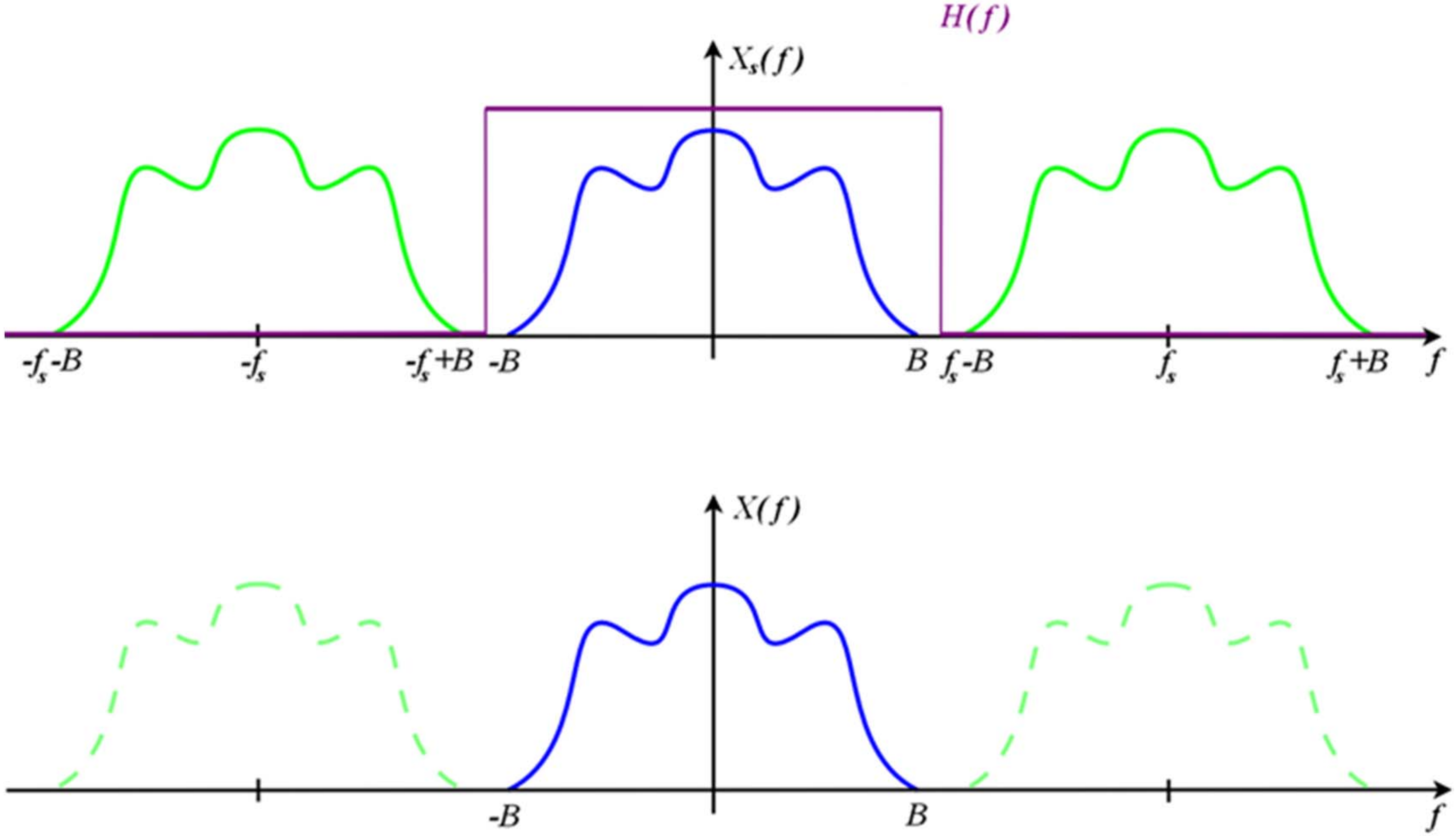
Sampling theorem

- Reconstruction



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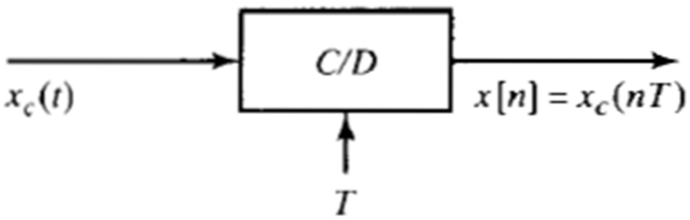
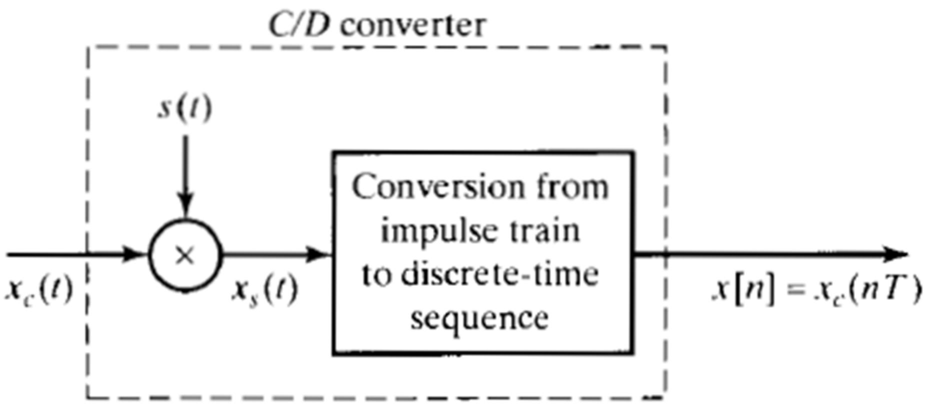
http://en.wikipedia.org/wiki/Nyquist%E2%80%93Shannon_sampling_theorem

Sampling theorem

- Illustration of impulse sampling



Block diagram representation of an ideal continuous-to-discrete-time (C/D) converter.



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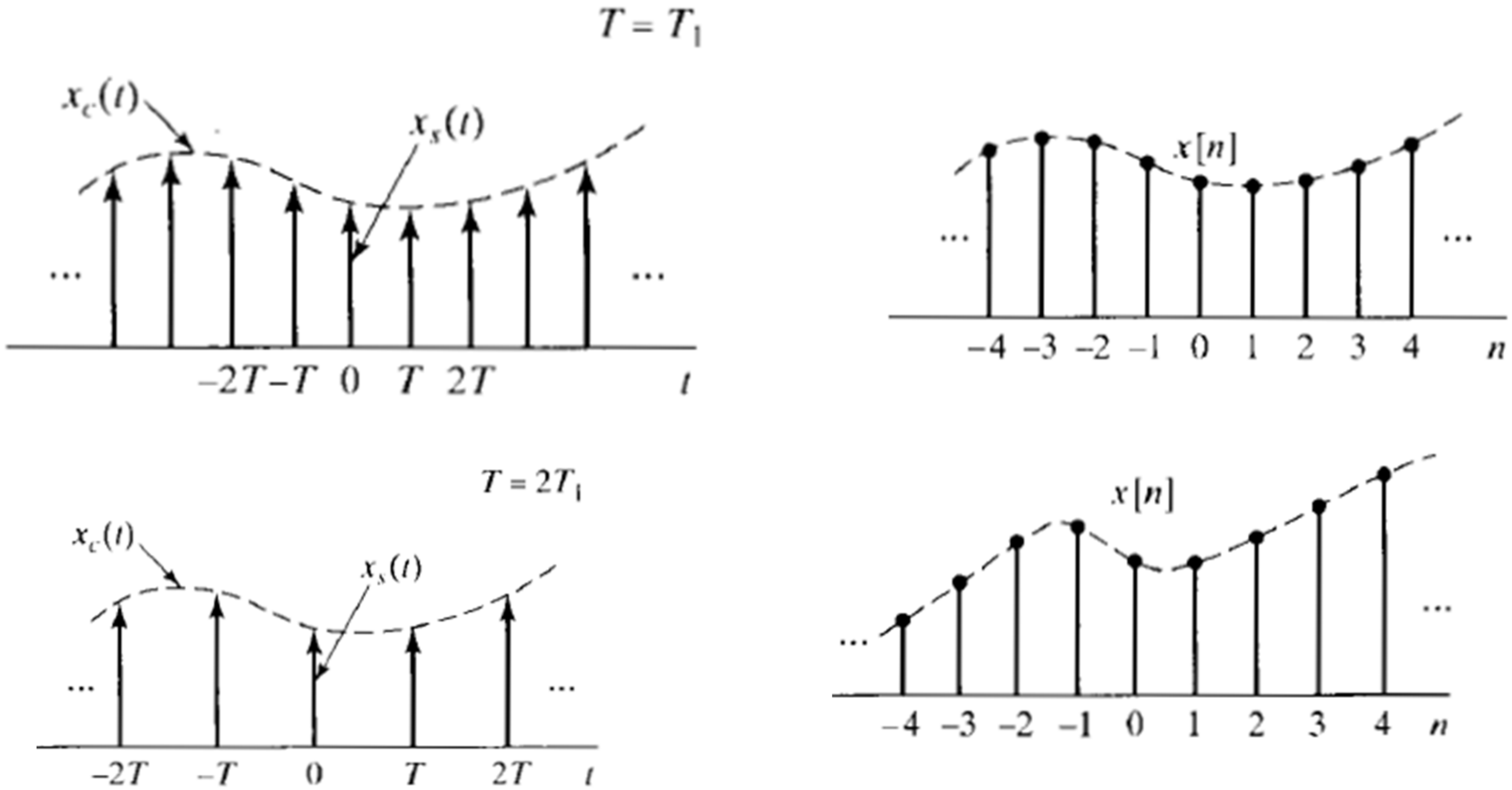
Sampling theorem

- And,



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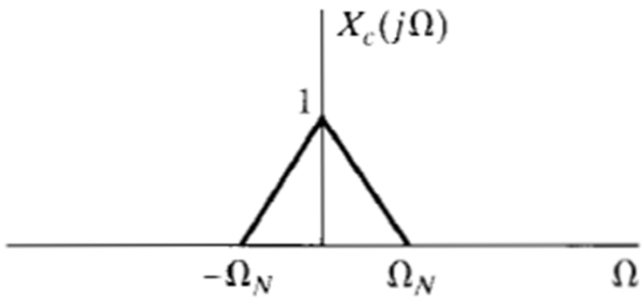
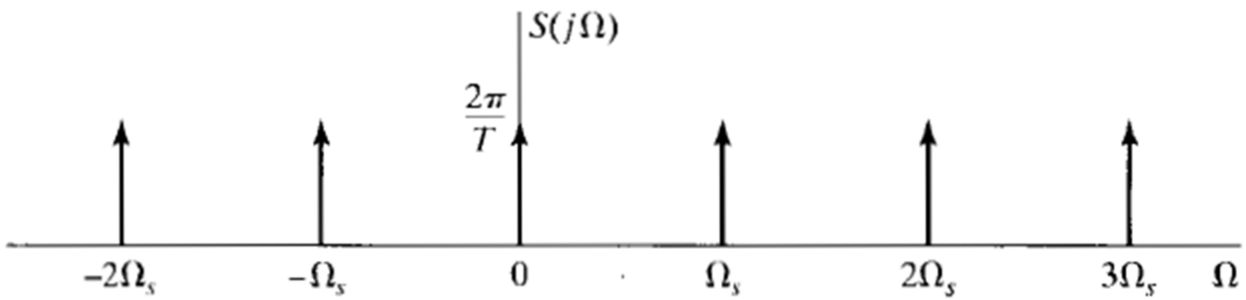
Sampling theorem

- Another view





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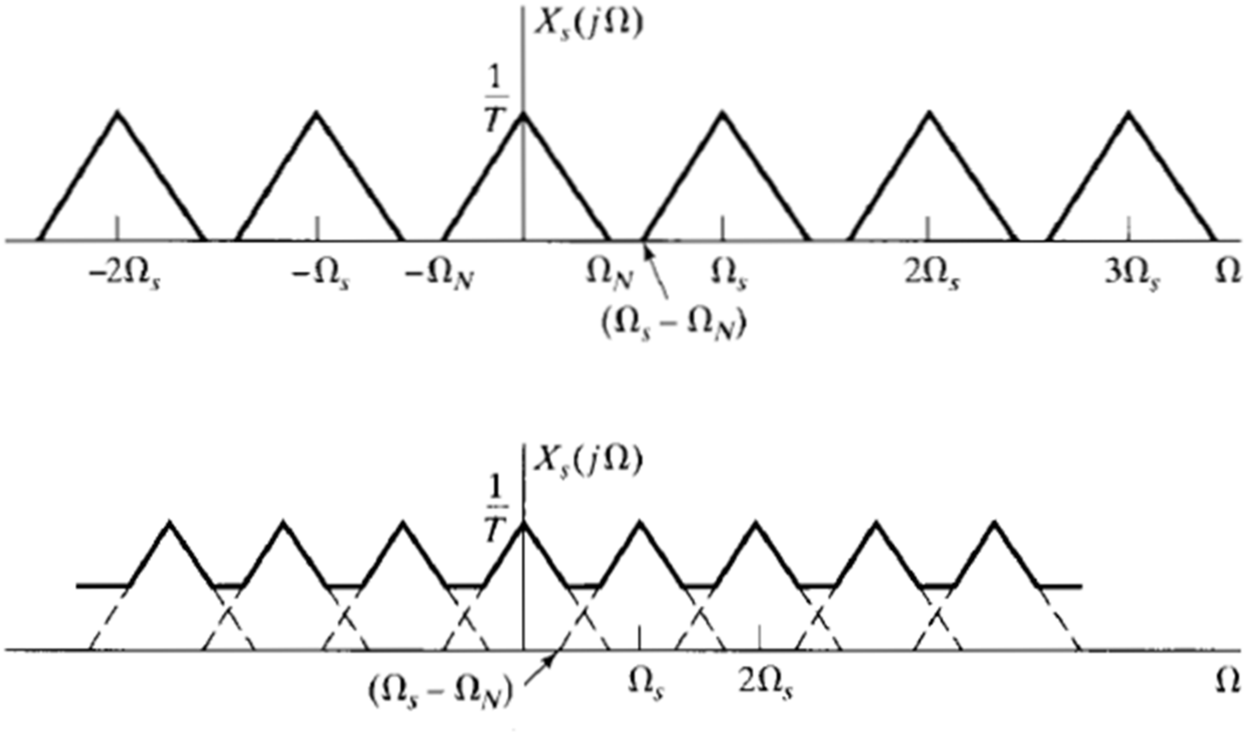
Sampling theorem

- An aliased spectrum



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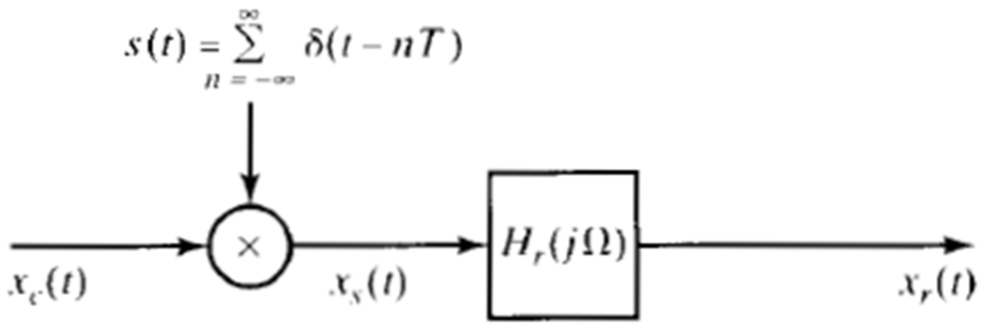
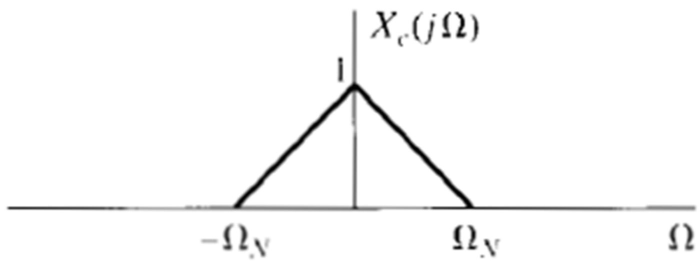
Sampling theorem

- Exact reconstruction





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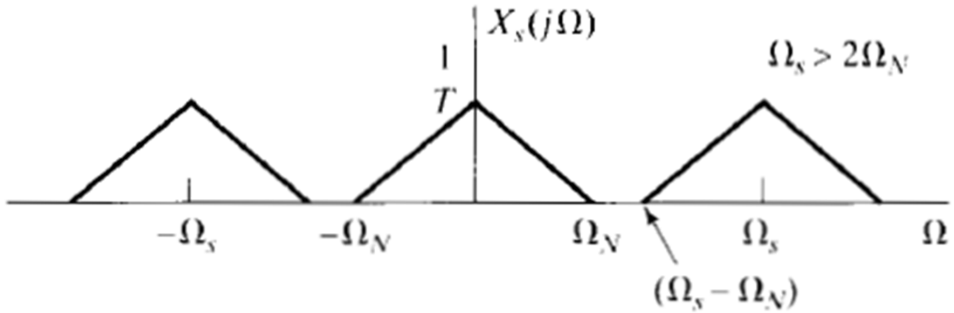
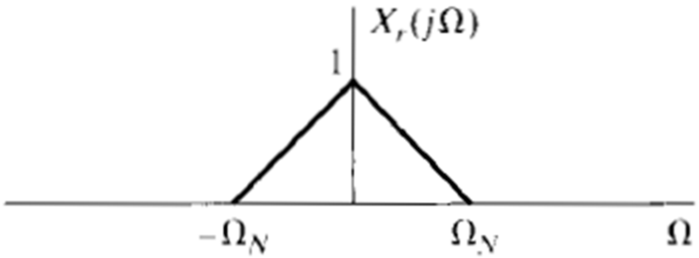
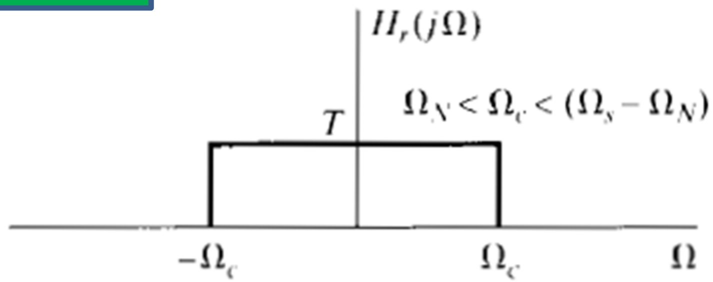
Sampling theorem

- We have that

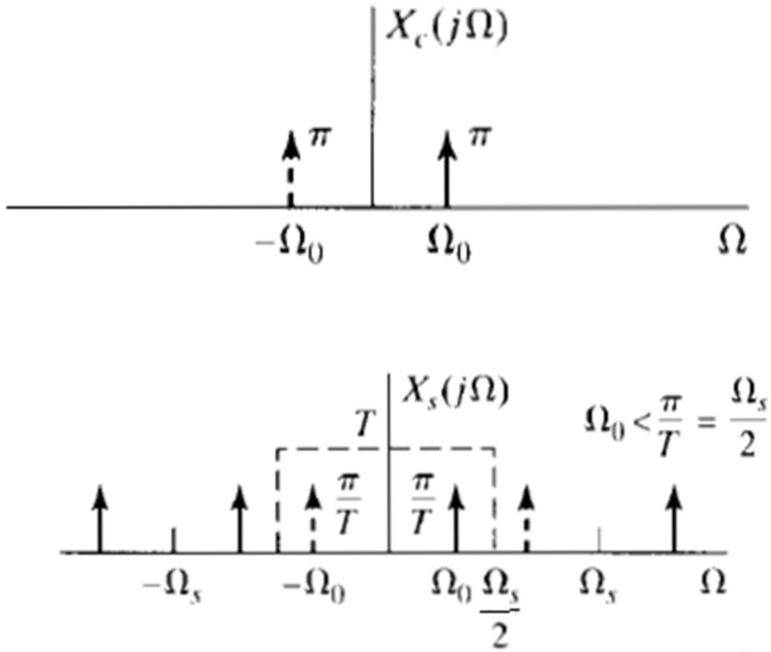






Sampling theorem

- Example: $x_c(t) = \cos \Omega_0 t$.



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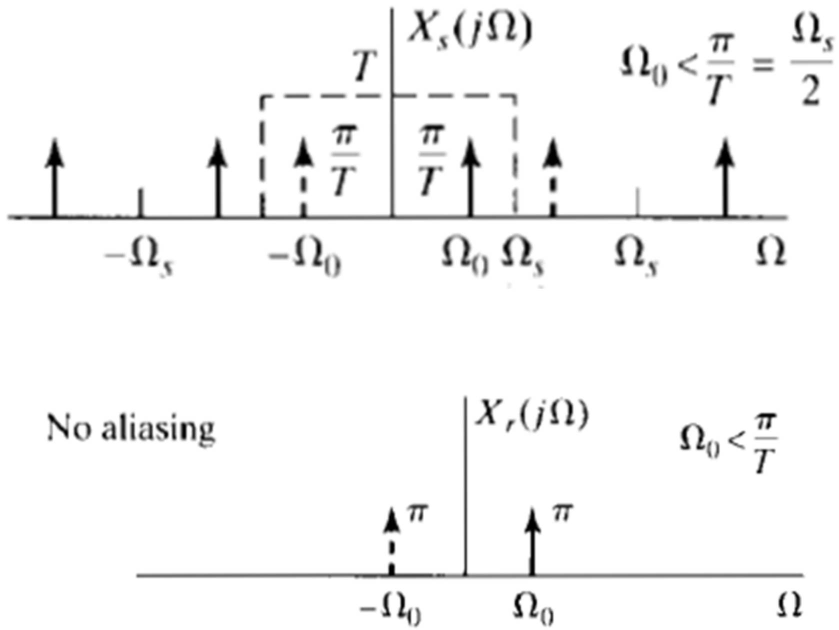
Sampling theorem

- Successful reconstruction



$$x_r(t) = \cos \Omega_0 t.$$

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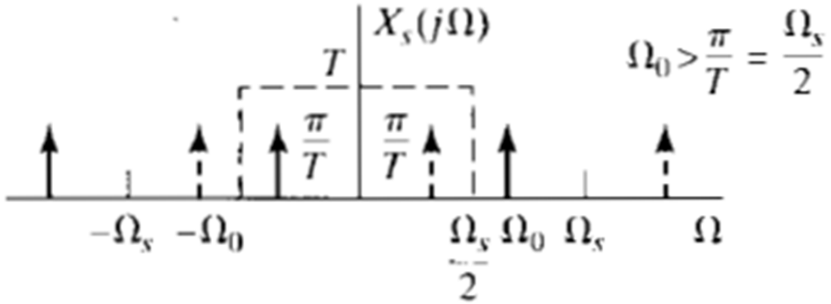
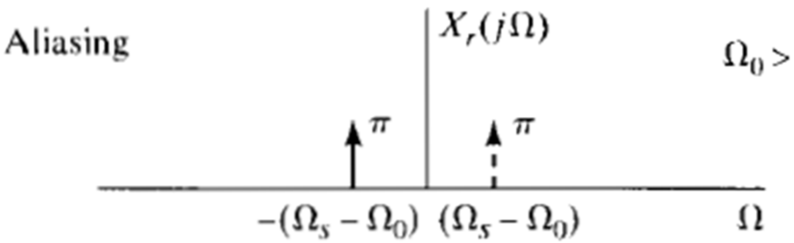
Sampling theorem

- Reconstruction with aliasing





$$x_r(t) = \cos(\Omega_s - \Omega_0)t$$

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Sampling theorem

- Remember that

$$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT).$$

  – And we used an ideal LPF for recovery; hence, the output is given by

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]h_r(t - nT).$$

  – Where $h_r(t)$ denotes the response of the filter

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Sampling theorem

- Cut-off frequency of the LPF
  - Between Ω_N and $\Omega_s - \Omega_N$
  - For cut-off frequency equal to Nyquest rate:

$$h_r(t) = \frac{\sin(\pi t/T)}{\pi t/T} \qquad \Omega_c = \Omega_s/2 = \pi/T$$

  - And

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T} \qquad x[n] = x_c(nT)$$

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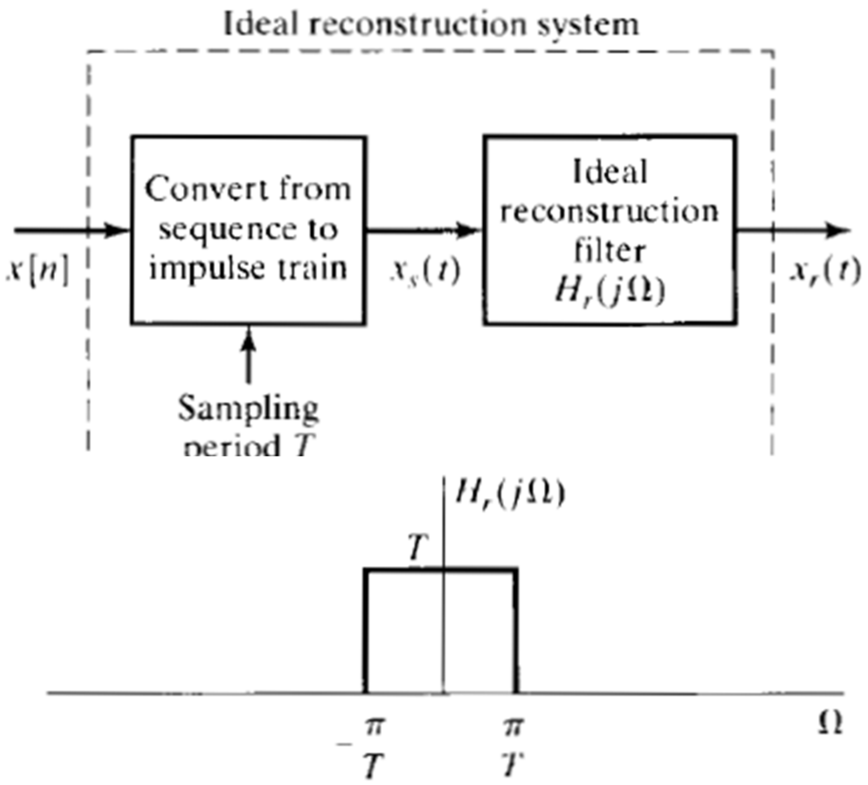
Sampling theorem

- Block diagram



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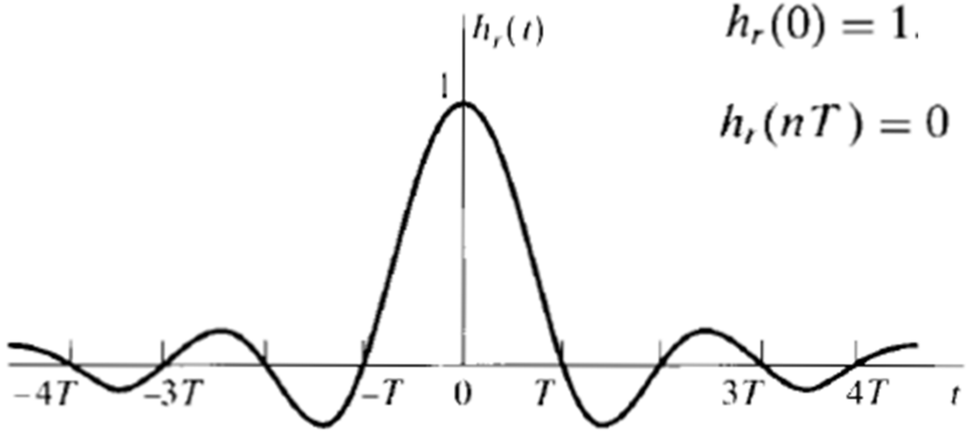
Sampling theorem

- Insight to time-domain recovery!
  - Impulse response of the reconstruction filter (sinc)



$$h_r(0) = 1.$$

$$h_r(nT) = 0 \quad \text{for } n = \pm 1, \pm 2, \ldots .$$

  - Hence, $x_r(mT) = x_c(mT)$ because $x_r(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T}$

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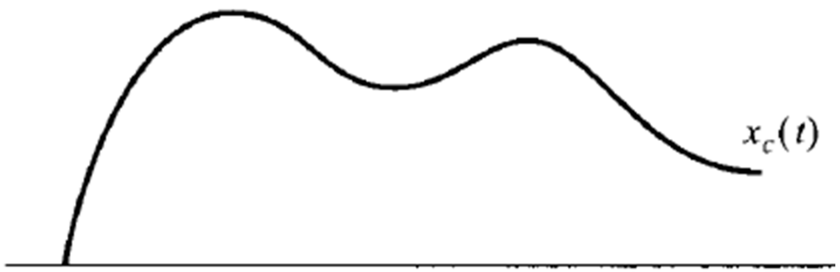
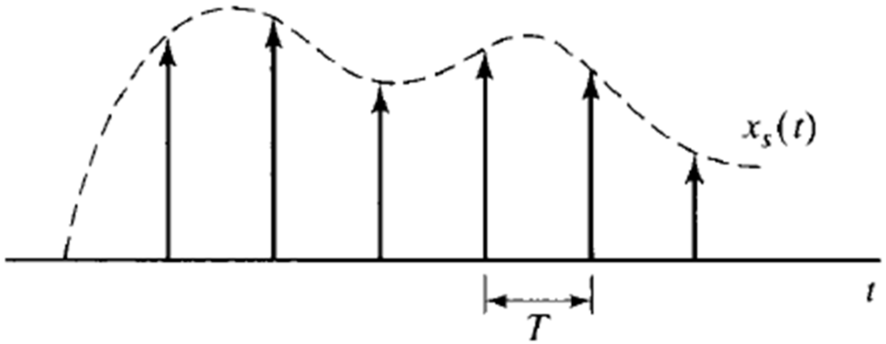
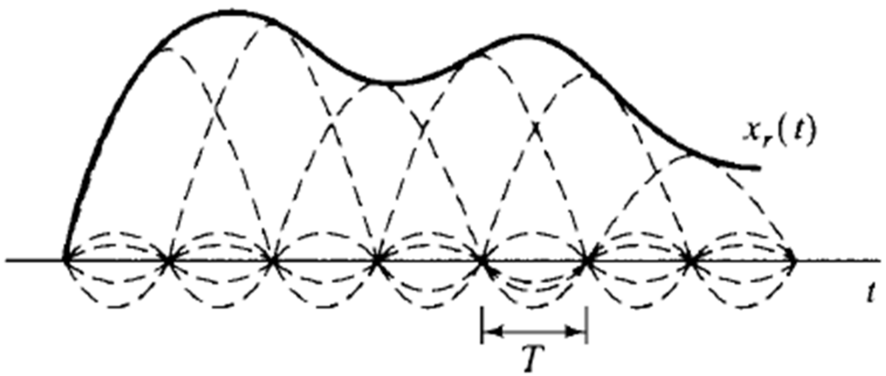
Sampling theorem

- Observe that







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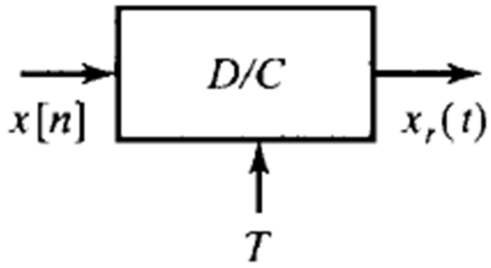
Sampling theorem

- Ideal interpolation
  - Sinc interpolation
  - Linear/quadratic interpolation
  - Qubic interpolation



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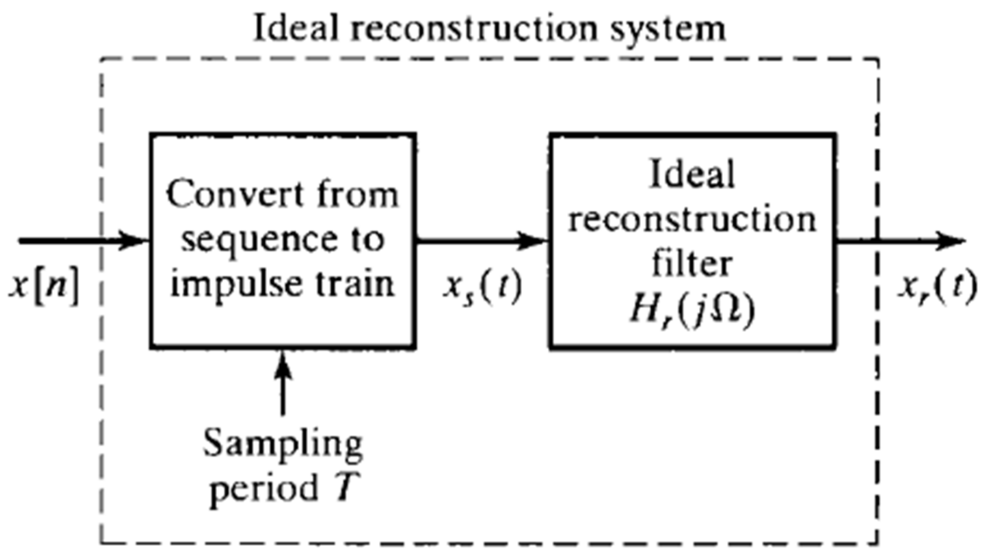
Sampling theorem

- Detailed diagram



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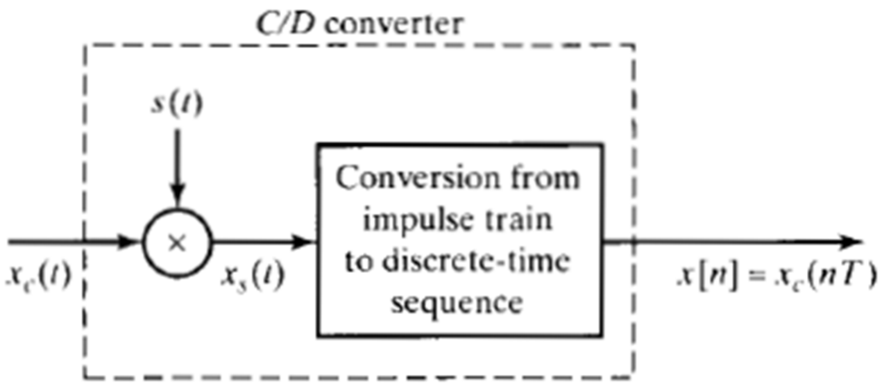
Sampling theorem

- Question: the relationship of the DTFT of the samples signal and the CTFT of the contiguous signal?



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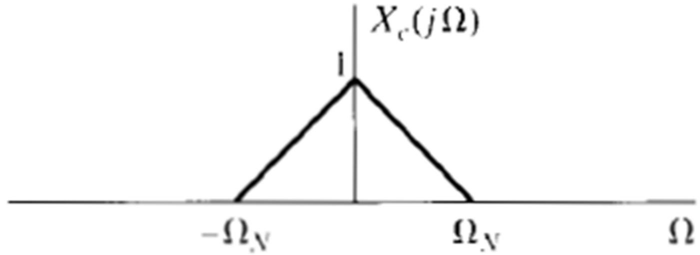
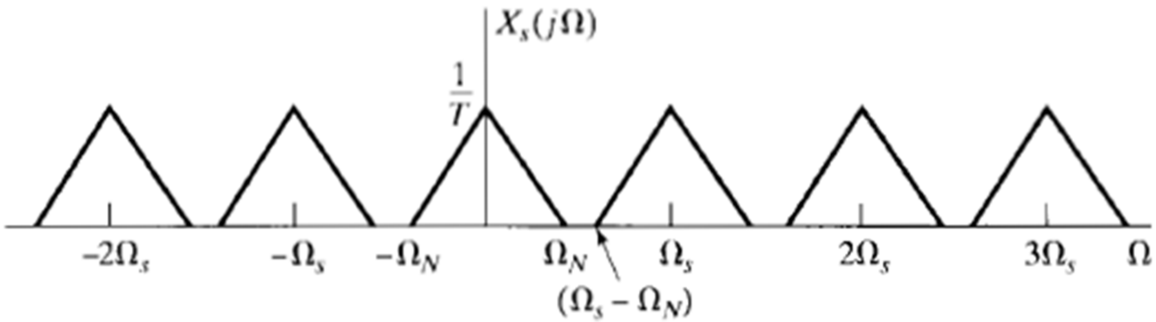
Sampling theorem

- Note that, as expected, the spectrum of $x_s(t)$ is periodic





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Sampling theorem

- Important

$$x_s(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\delta(t - nT)$$

 – Applying CTFT yields

$$X_s(j\Omega) = \sum_{n=-\infty}^{\infty} x_c(nT)e^{-j\Omega Tn}.$$

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Sampling theorem

- On the other hand,

$$x[n] = x_c(nT)$$

  - With DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}.$$

  - Therefore,

$$X_s(j\Omega) = X(e^{j\omega})|_{\omega=\Omega T} = X(e^{j\Omega T}).$$

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Sampling theorem

- Notes:
  - Discrete-time frequency
    - Normalized
    - Radian
    - Maximum frequency
  - Continuous-time frequency
    - Radian/sec or Hz

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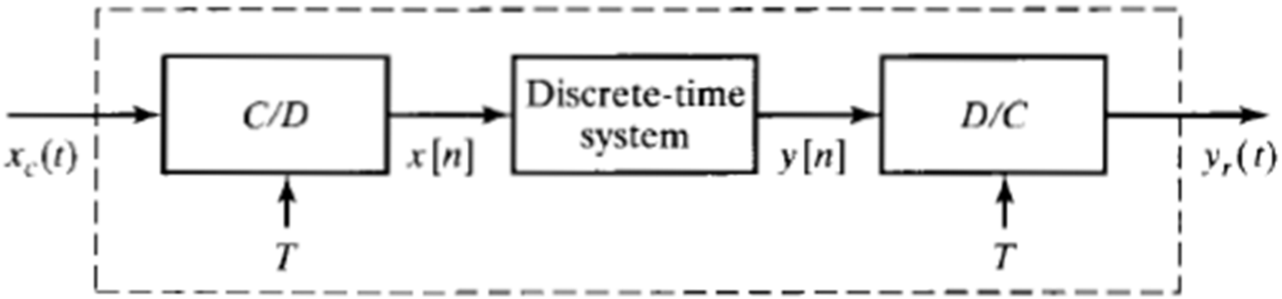
Sampling theorem

- Discrete-time processing of the continuous-time signals



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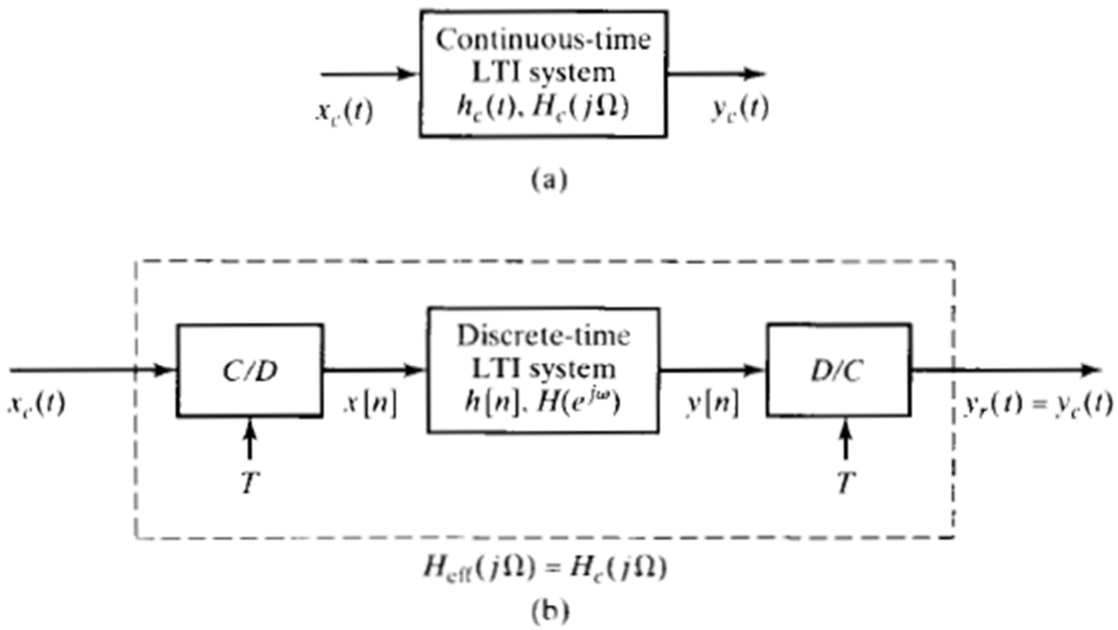
Sampling theorem

- Diagram



(a) Continuous-time LTI system. (b) Equivalent system for bandlimited inputs.

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