Digital Signal Processing (DSP)

Fall 2014

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DIGITAL SIGNAL PROCESSING (DSP)

Lecture 8
Filter Design

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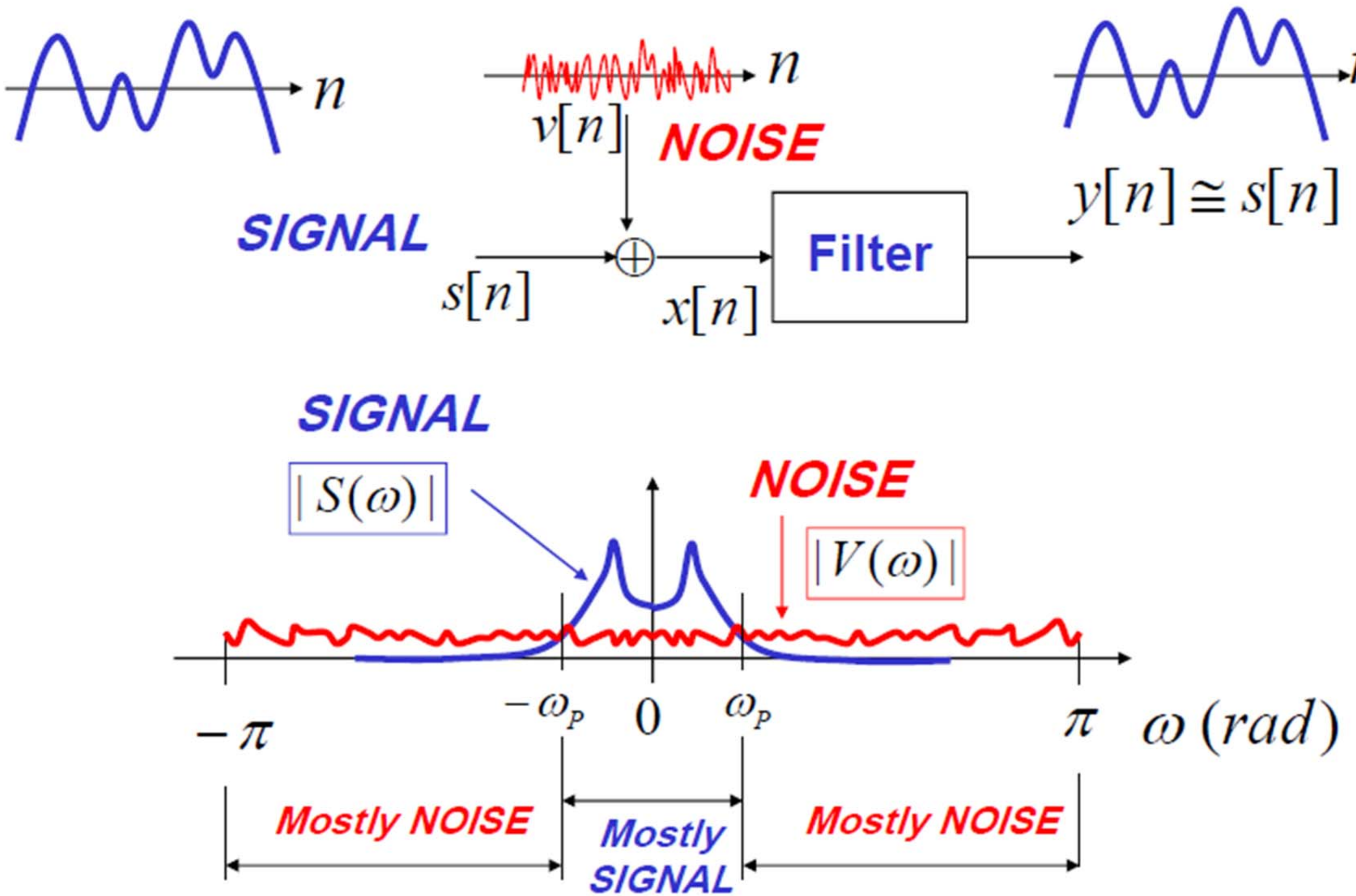
Most of the materials from the book "discrete-time signal processing" by Oppenheim.

Filter Design

- Ideal LPF
  - Non-causal
  - Unstable
  - Can not be realized!
- We are going to design realizable filter satisfying a given set of specifications
- Analog filters
- Digital filters

Filter Design



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http://www.faculty.nps.edu/...Filters/slides/2-Filter-Design....

Filter Design

- Design problem: a frequency response (system function) is given. Compute the impulse response/coefficients of the realizable system
- Causal system
- Stable system
- Sometimes linear phase
- Approximate design

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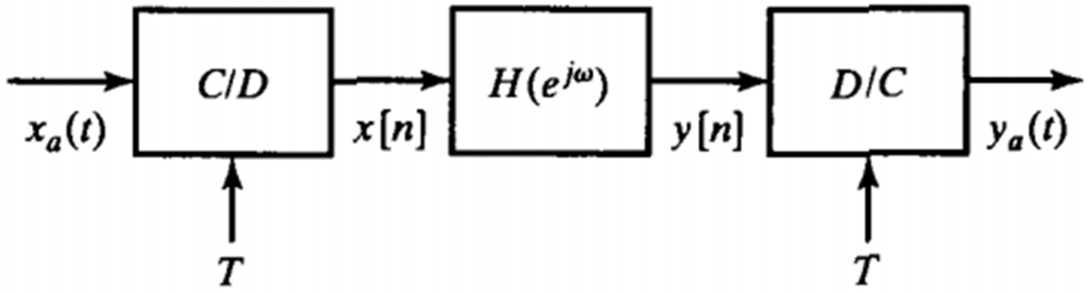
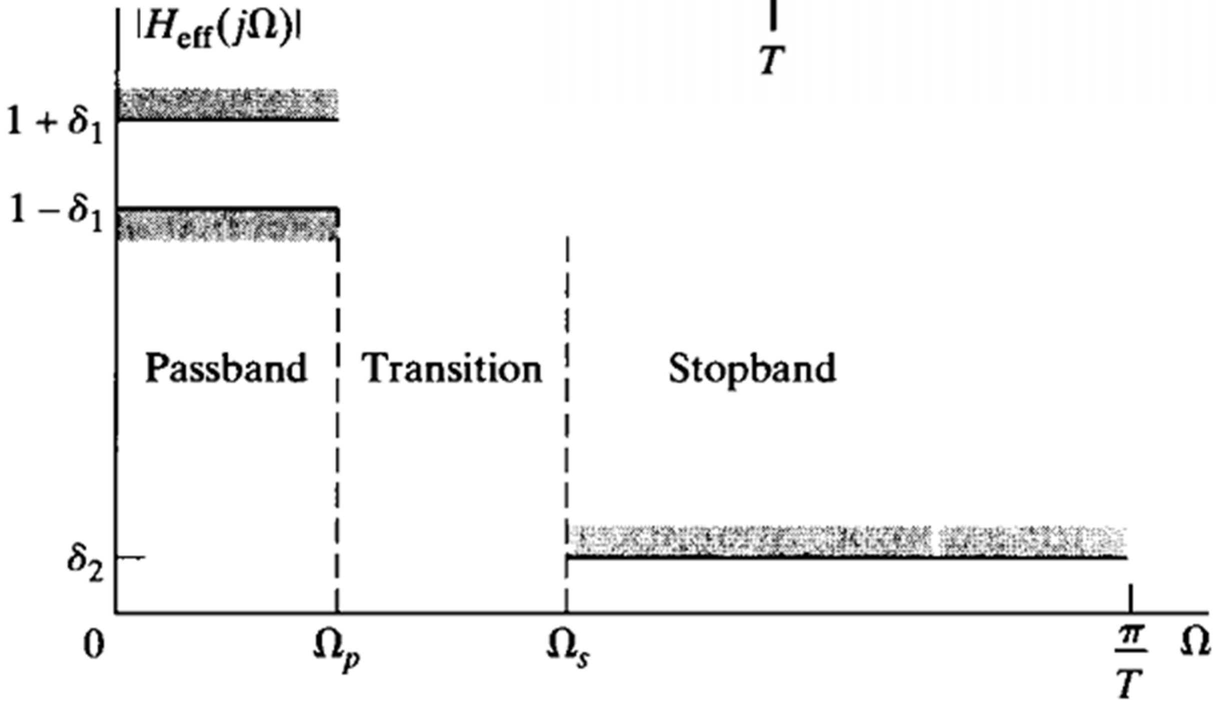
Filter Design

- Illustration





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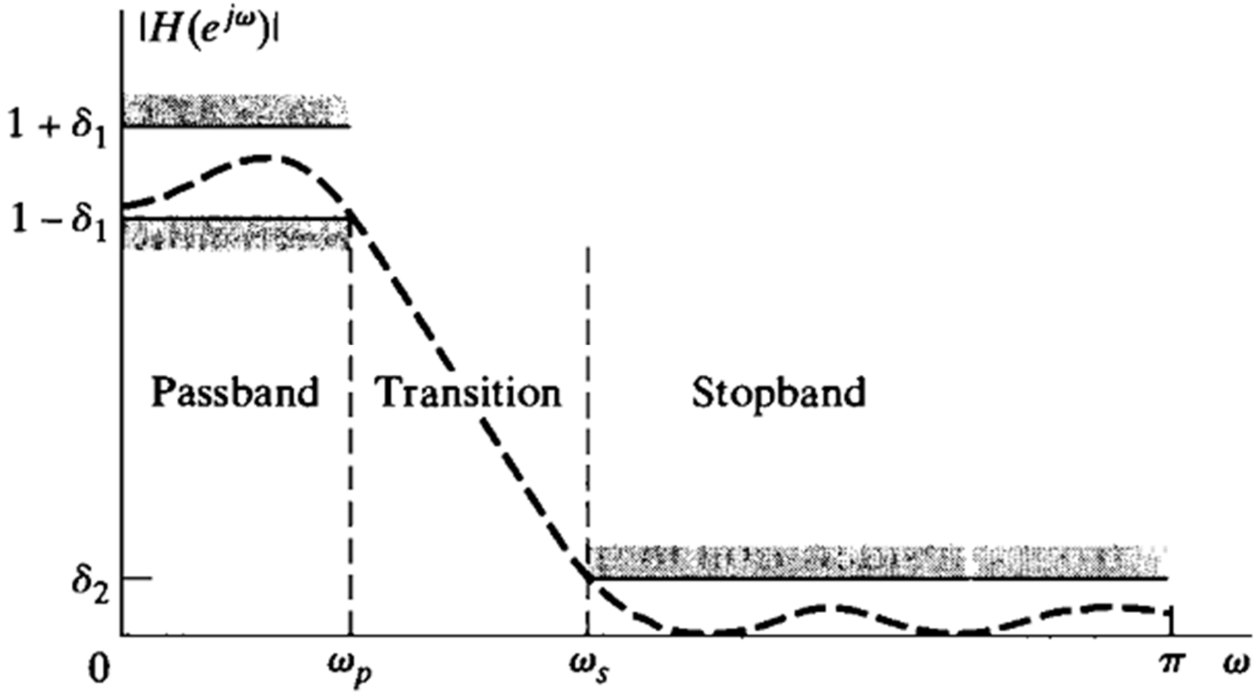
Filter Design

- Cont.

$$H(e^{j\omega}) = H_{\text{eff}}\left(j\frac{\omega}{T}\right), \qquad |\omega| < \pi.$$



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Filter Design

- A typical set of values

$$\delta_1 = 0.01,$$
$$\delta_2 = 0.001,$$
$$\Omega_p = 2\pi(2000),$$
$$\Omega_s = 2\pi(3000).$$

- Usually given in dB

$$\begin{aligned}
&\text{ideal passband gain in decibels} &&= 20\log_{10}(1) &&= 0 \text{ dB}\\
&\text{maximum passband gain in decibels} &&= 20\log_{10}(1.01) &&= 0.086 \text{ dB}\\
&\text{maximum stopband gain in decibels} &&= 20\log_{10}(0.001) &&= -60 \text{ dB}
\end{aligned}$$

Filter Design

- Design constraints
  - Passband

$$(1-\delta_1) \leq |H(e^{j\omega})| \leq (1+\delta_1), \qquad |\omega| \leq \omega_p,$$

  - Stopband

$$|H(e^{j\omega})| \leq \delta_2, \qquad \omega_s \leq |\omega| \leq \pi$$

  - Transition band

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Filter Design

- FIR: finite impulse response
- IIR: infinite impulse response
- Remark: generalized linear phase
  - Linear phase

$$\theta(\omega) = \angle H(e^{j\omega}) = -c\omega \qquad H(e^{j\omega}) = |H(e^{j\omega})|e^{-j\omega c}$$

  - generalized linear phase

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega c + jb}$$

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Filter Design

- Selection
  - FIR: causal+ stable+ GLP
  - IIR: causal+ stable+ satisfying spec. with lower complexity

- A rich literature on analog filter design
  - Butterworth
  - Chebyshev

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Filter Design

- Idea: digital filter (IIR) design via CT filters
  - 1) Design CT filter h(t) from given frequency response
  - 2) Employ impulse invariance to obtain the DT version of the filter

- Other methods
  - Bilinear transform, etc.

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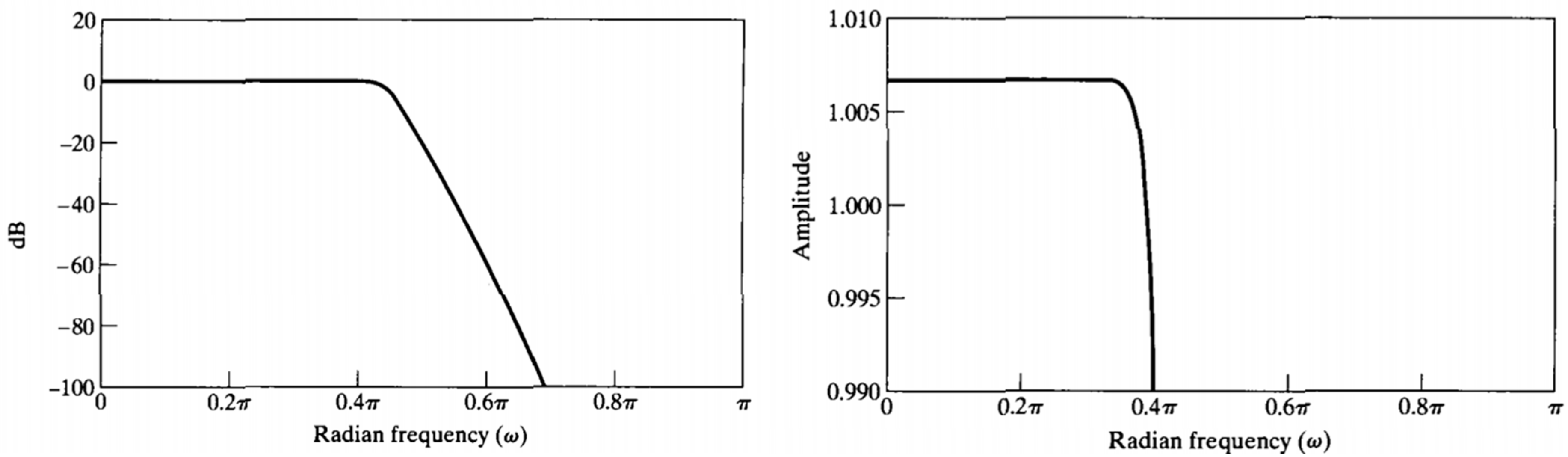
Filter Design

- Butterworth



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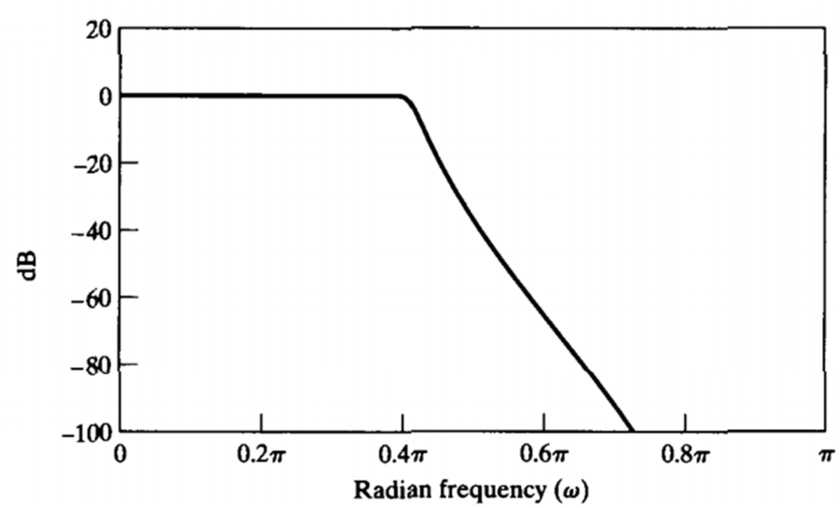
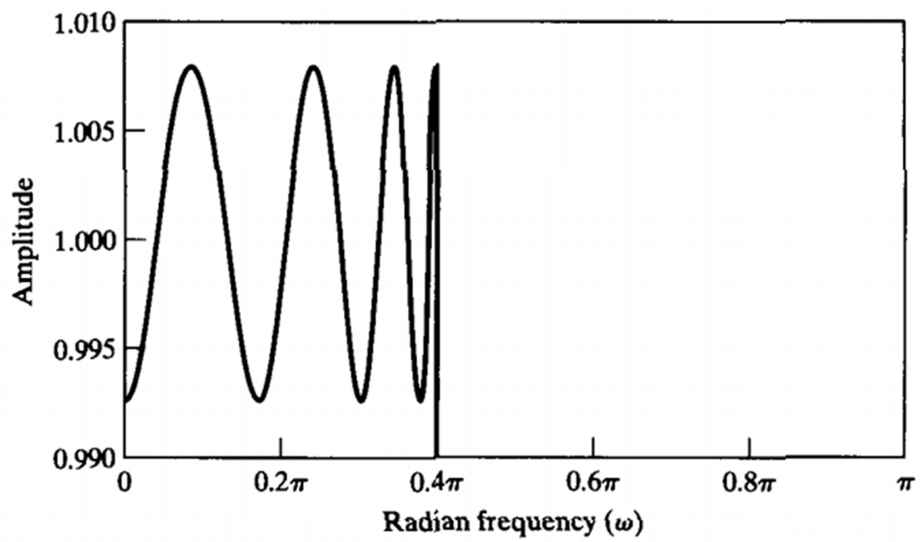
Filter Design

- Chebyshev (type I): ripple in passband





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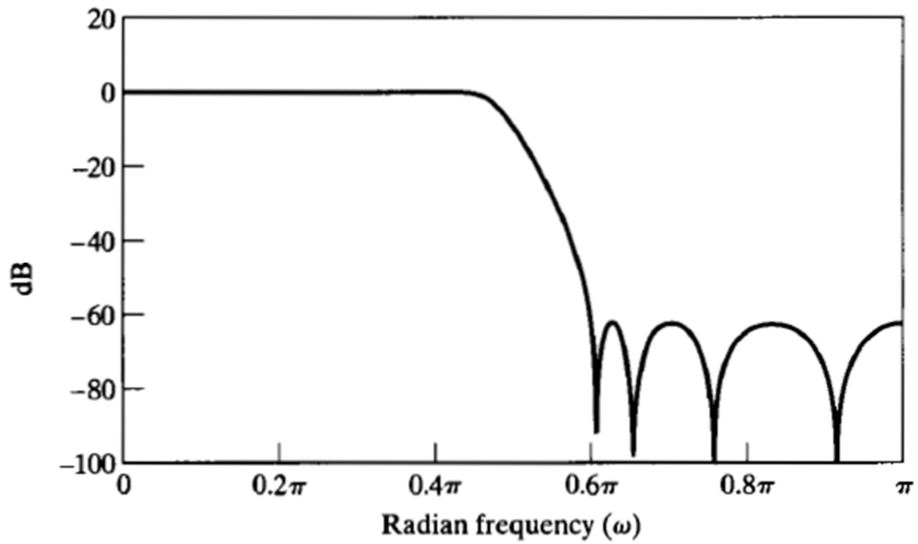
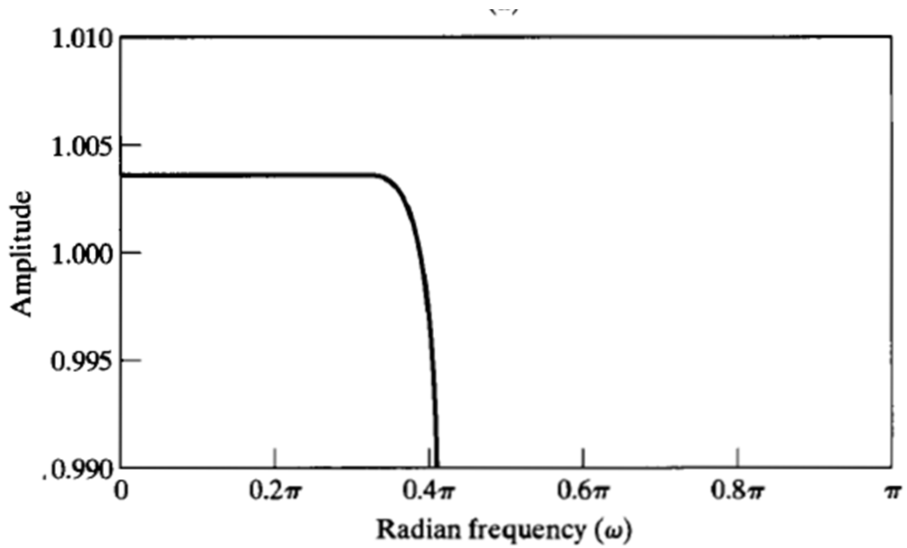
Filter Design

- Chebyshev (type II): ripple in stopband





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Filter Design

- FIR filter design: our main focus
  - Arising in discrete-time implementation
  - Approximation of the response in discrete domain
  - (generalized) linear phase
- Design methods
  - Design by windowing
  - Design by optimal approximation

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Filter Design

- Windowing method
  - Given the ideal response

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n]e^{-j\omega n},$$

  - And hence

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

  - Usually non-causal/unstable

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Filter Design

- The idea is to truncate the impulse response

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n]e^{-j\omega n},$$

- Actually, the FIR approximation is given by

$$h[n] = \begin{cases} h_d[n], & 0 \le n \le M, \\ 0, & \text{otherwise.} \end{cases}$$

  - Stable
  - Causal

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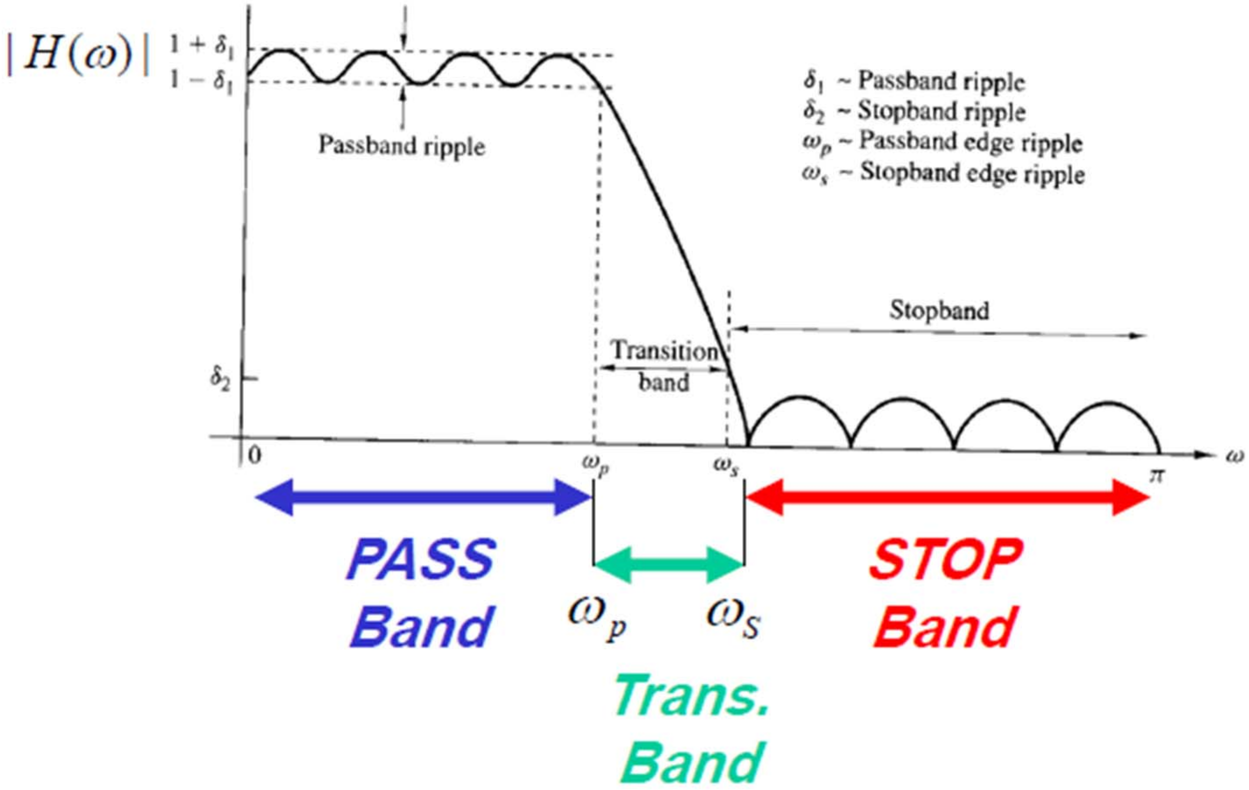
Filter Design

- Example: LPF



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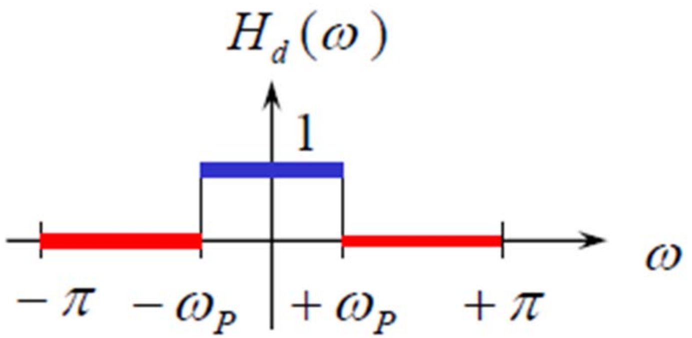
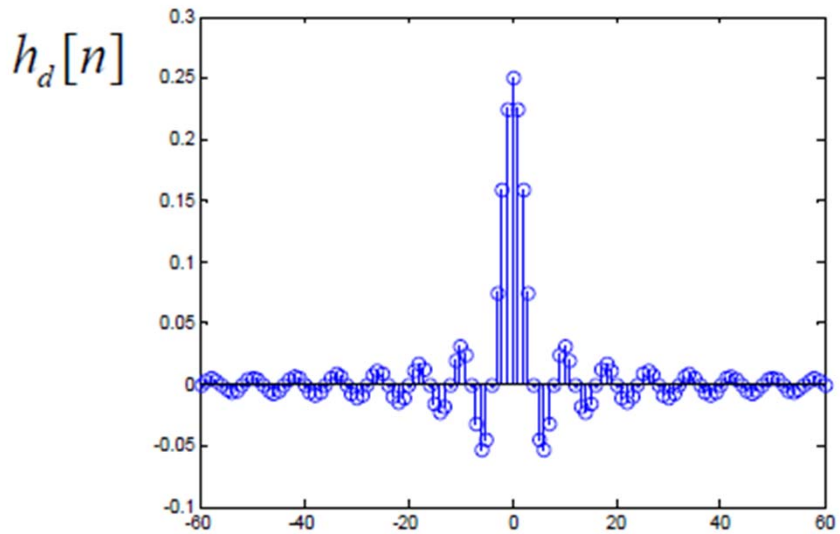
Filter Design

- Ideal form





$$h_d[n] = \frac{\sin(\omega_P n)}{\pi n} = \frac{\omega_P}{\pi}\,\mathrm{sinc}\left(\frac{\omega_P}{\pi} n\right)$$

$$H_d(\omega) = \sum_{n=-\infty}^{+\infty} h_d[n] e^{-j\omega n}$$

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Filter Design

– Noting that

$$\lim_{n \to \pm\infty} h_d[n] = 0$$

– We use the following approximation

$$H_d(\omega) \cong \sum_{n=-L}^{+L} h_d[n] e^{-j\omega n}$$

– And select the below filter to be causal

$$h[n] = h_d[n - L]$$

Filter Design

- Therefore, via employing truncating:

$$H(\omega) = \sum_{n=0}^{2L} h[n] e^{-j\omega n} = \sum_{n=0}^{2L} h_d[n-L] e^{-j\omega n} = \left(\sum_{n=-L}^{L} h_d[n] e^{-j\omega n} \right) e^{-j\omega L} \cong H_d(\omega) e^{-j\omega L}$$

actual ***desired***

- and

Magnitude: $|H(\omega)| \cong |H_d(\omega)|$

Phase: $\angle H(\omega) = \angle H_d(\omega) - \omega L$

$= -\omega L$ ***in the passband***

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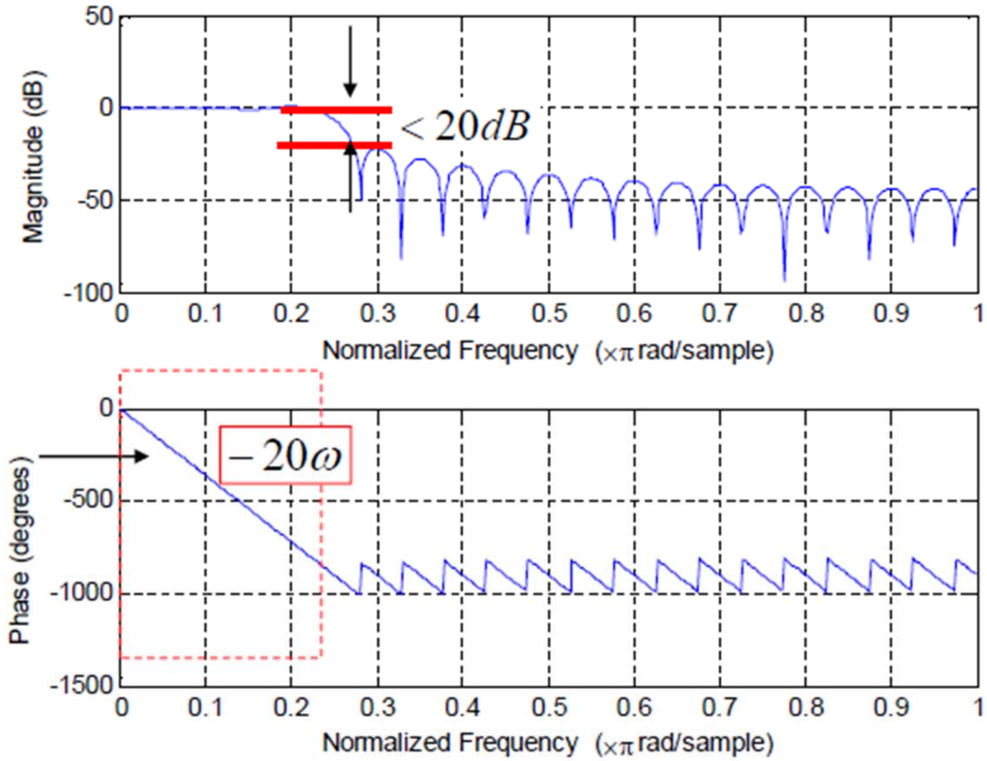
Filter Design

- Example (cont.)

$$\omega_p = \pi / 4 \qquad L = 20, \quad N = 40$$



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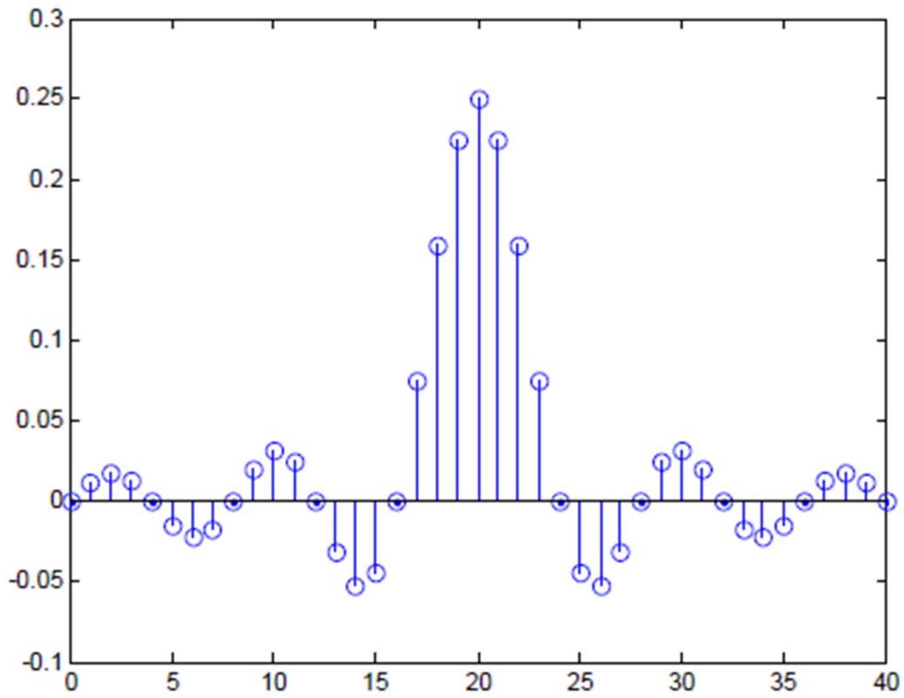
Filter Design

- And in the time domain
  - Causal
  - Stable



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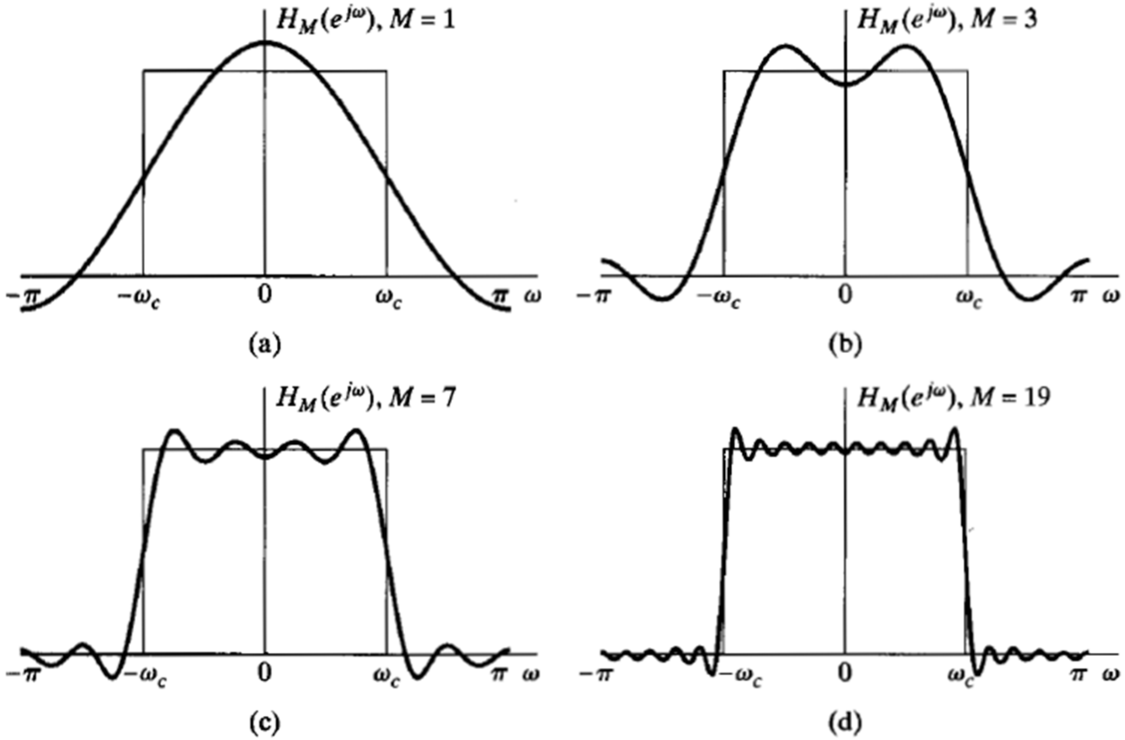
Filter Design

- Remember: Gibbs phenomenon

$$H_M(e^{j\omega}) = \sum_{n=-M}^{M} \frac{\sin \omega_c n}{\pi n} e^{-j\omega n}$$



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Filter Design

- The name of the method: windowing!!
  - Generally, we can use a window function w[n]:

$$h[n] = h_d[n]w[n],$$

  - Win the previous case (rectangular window)

$$w[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise.} \end{cases}$$

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Filter Design

- What happens by using window function?

$$h[n] = h_d[n]w[n],$$

  - In frequency domain

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta})W(e^{j(\omega-\theta)})d\theta$$

  - The output spectrum is affected by the spectral content of the window

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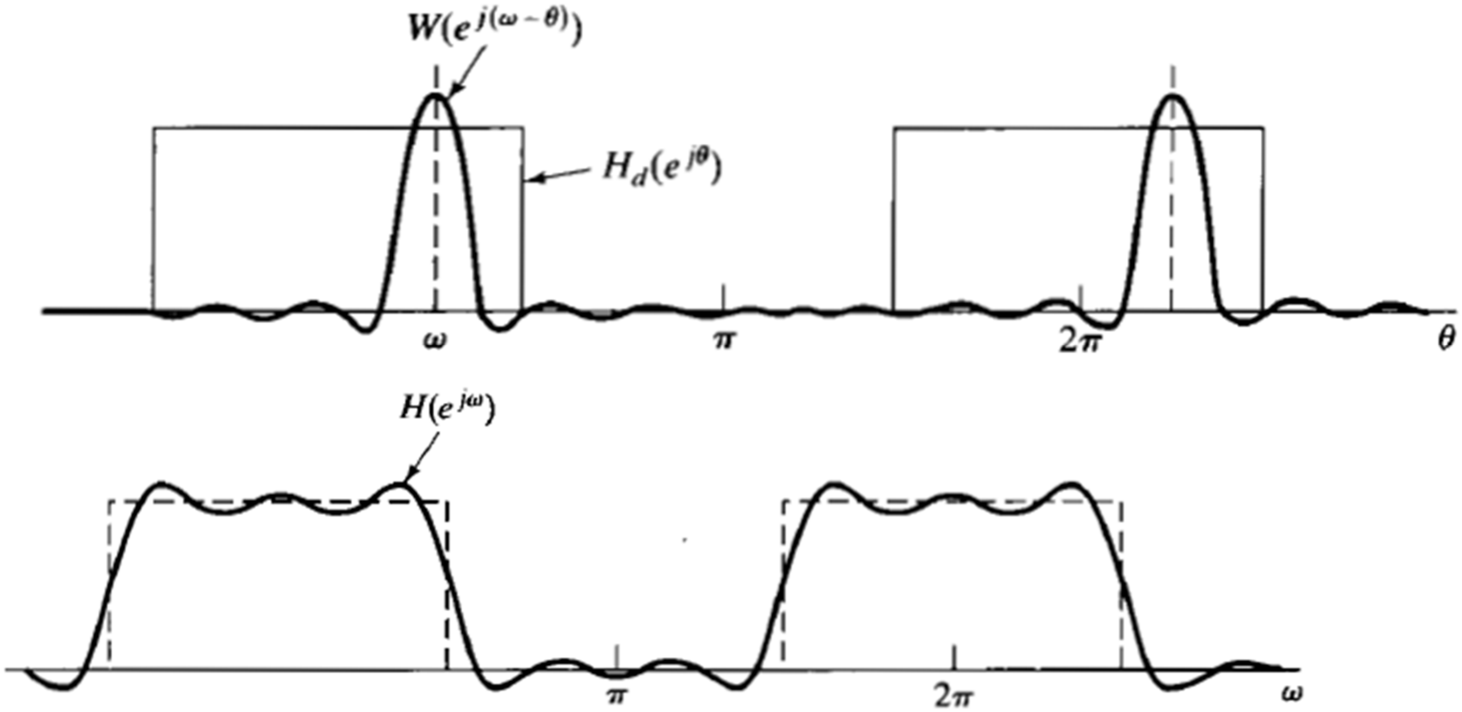
Filter Design

- Illustration



Filter Design

- Special case: *w[n]=1* for all *n*
  - The resulting DTFT is given by impulse train with period; hence

$$H(e^{j\omega}) = H_d(e^{j\omega})$$

  - (*)Observation: narrowband window functions are desired
    - Approximately reproducing the desired ideal filter

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Filter Design

- (**)Implementation issue: shrinking the window function to reduce computational burden as much as possible
- Items * and ** are conflicting requirements

- Illustration of the conflict for rectangular window

$$W(e^{j\omega}) = \sum_{n=0}^{M} e^{-j\omega n} = \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}} = e^{-j\omega M/2} \frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}$$

  - Linear phase

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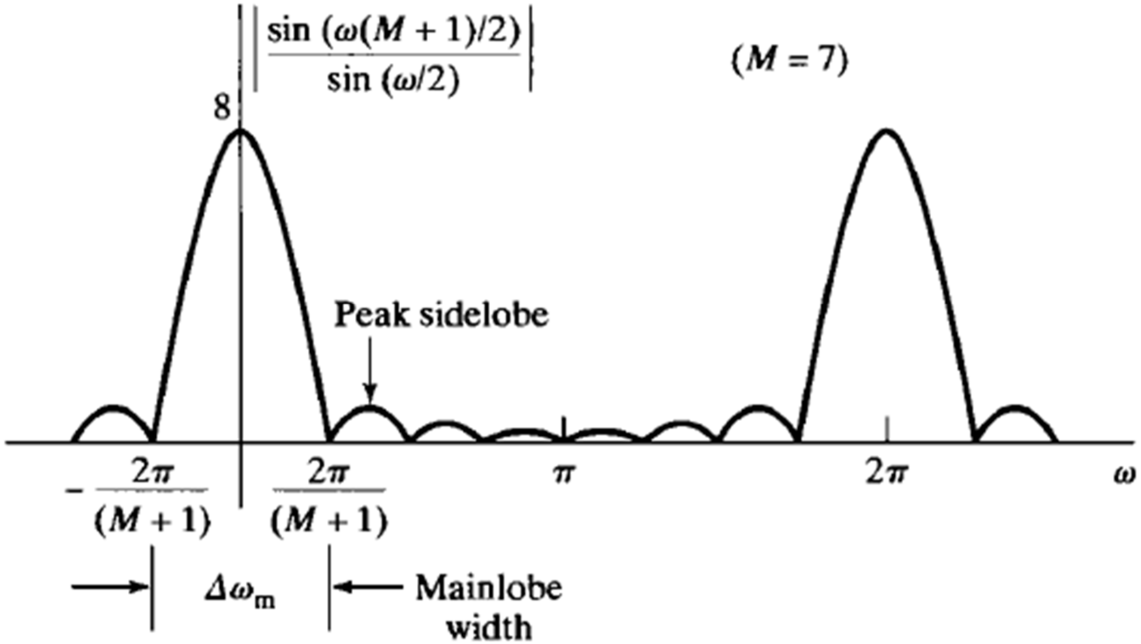
Filter Design

- Spectrum of the rectangular window



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Filter Design

- Observations from the figure
  - Larger *M* leads to narrower mainlobe
    - The region between the first zero-crossing of the sides

$$\Delta_{\omega_m} = 4\pi/(M+1)$$

  - Larger *M* is associated with higher computational load
  - The energy of the sidelobes do not change with *M*
  - Oscillation due to sidelobes: Gibbs phenomenon

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Filter Design

- The choice of the window type is important
  - Mainlobe width: sharpness
  - Sidelobes energy: oscillation

- For a given length of the window, various shapes can be used
  - Example: smoothing the rectangular window for sidelobe reduction

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Filter Design

- Window selection/design
  - Commonly used window functions

Rectangular

$$w[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Bartlett (triangular)

$$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2, \\ 2 - 2n/M, & M/2 < n \le M, \\ 0, & \text{otherwise} \end{cases}$$

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- Common window (cont.)

Hanning

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

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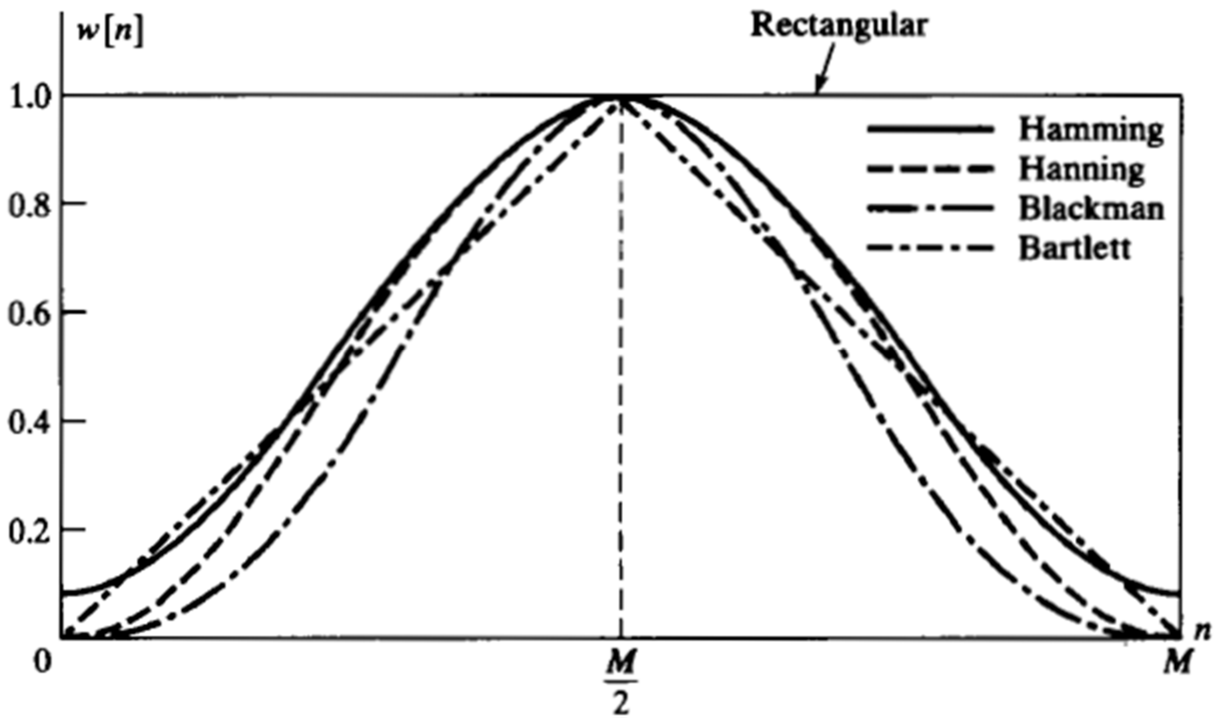
Filter Design

- Time domain behavior



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Filter Design

- Application
  - FIR filter design
  - Spectral analysis

- Properties
  - Simple functional form
    - Computation
  - Spectra are concentrated around zero frequency

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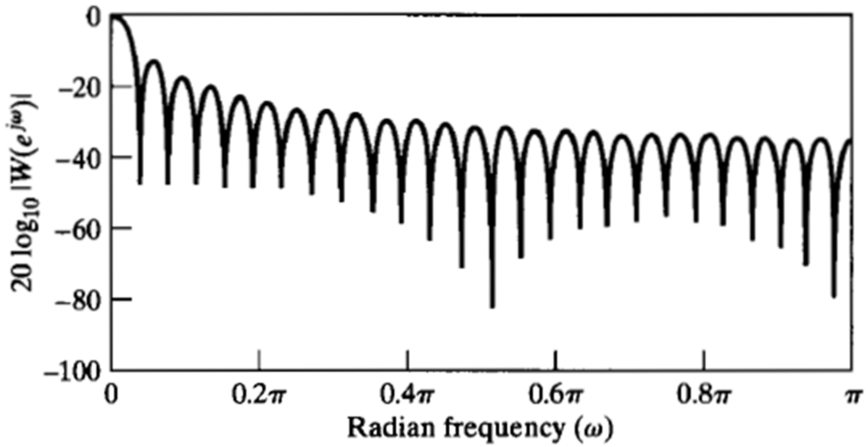
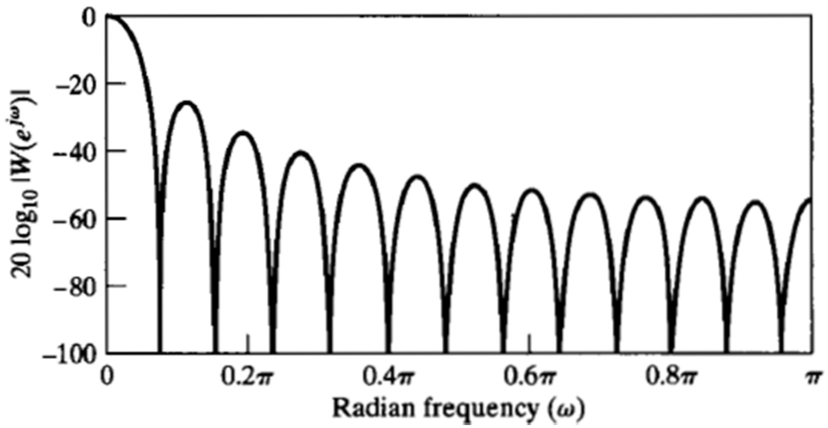
Filter Design

- Spectra



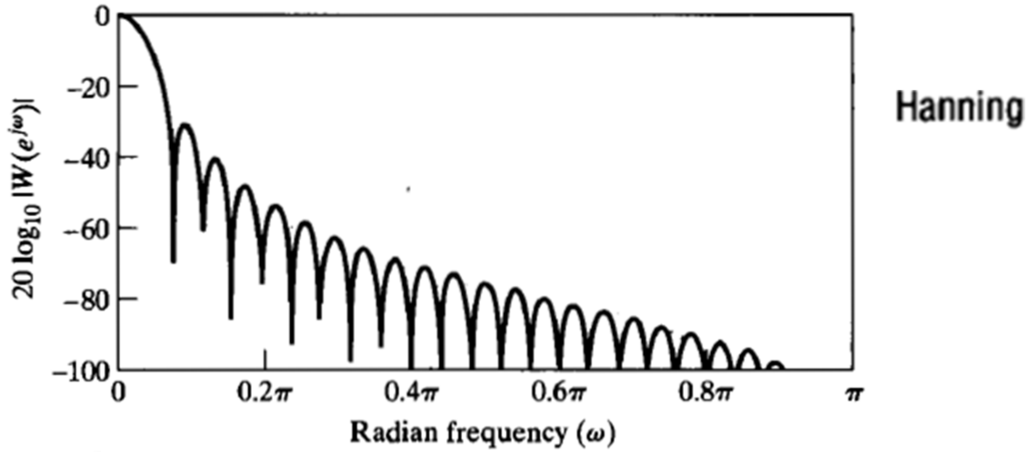
Rectangular.

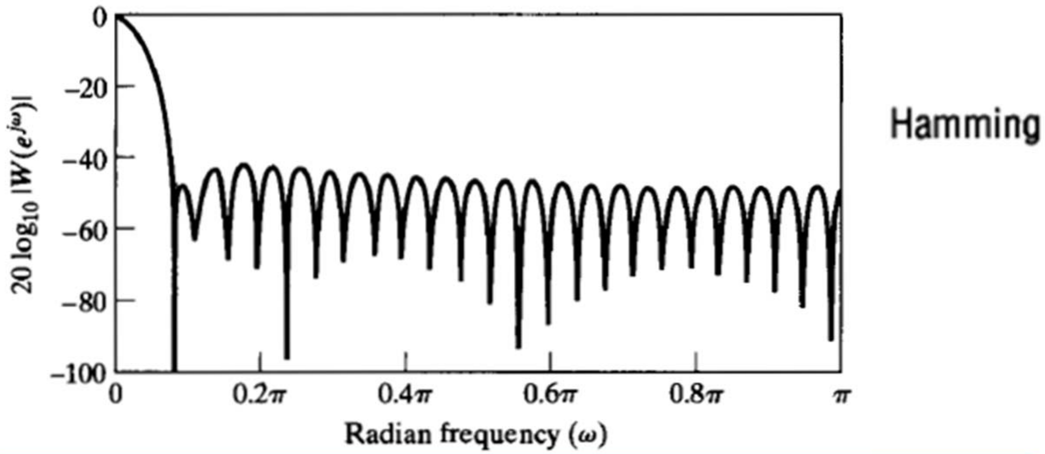


Bartlett

Filter Design

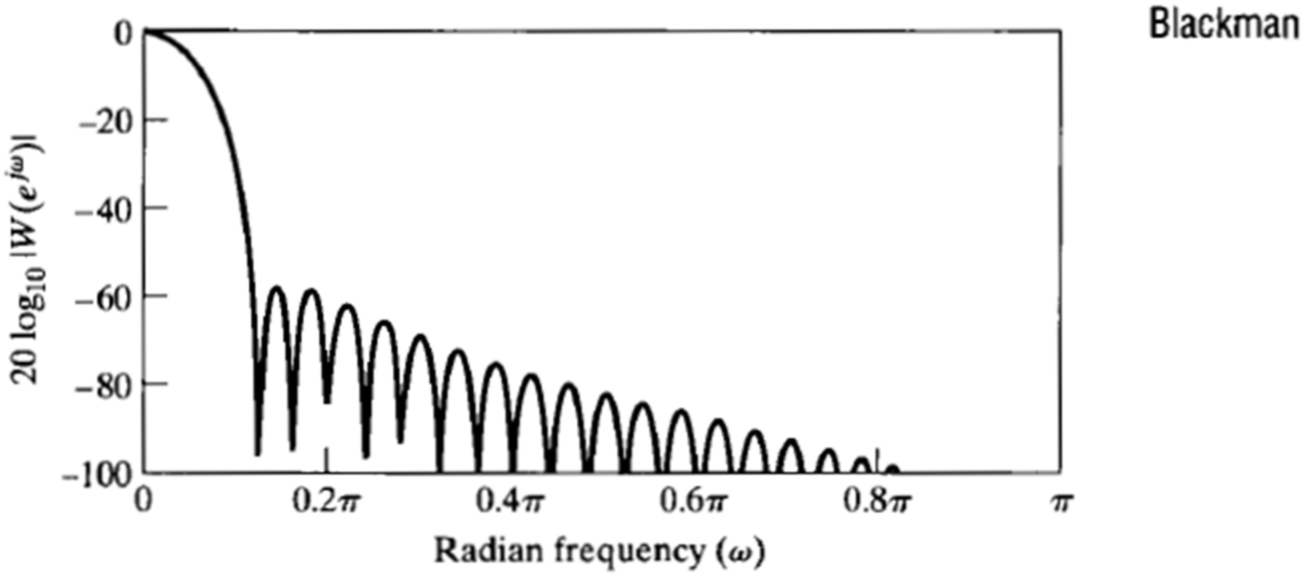
- Spectra





Filter Design

- Spectra



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Filter Design

- Summary of windows

TABLE COMPARISON

Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe
Rectangular	−13	$4\pi/(M+1)$
Bartlett	−25	$8\pi/M$
Hanning	−31	$8\pi/M$
Hamming	−41	$8\pi/M$
Blackman	−57	$12\pi/M$

- Cost of lower sidelobe level: wider mainlobe

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