Radar Waveform Design for Detection Performance Improvement

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Abstract

In this dissertation, we study the problem of waveform design for improvement of detection performance in radar systems. To this end, for single-input single-output (SISO) systems, we consider the effect of the signal-dependent interference at receive side (i.e., clutter) and the fact that Doppler shift of targets are often unknown at the transmit side. The raised design problems (for various situation) are non-convex and in some cases, belong to classes of NP-hard problems. We devise several methods to tackle the design problems and extend the proposed algorithms to the case of constrained design (peak-to-average-power-ratio (PAR) or similarity constraint). In case of multiple-input multiple-output (MIMO) systems, we derive the exact theoretical performance expressions of the optimal detector in Gaussian interference. Due to the complicated forms of the expressions, we use the most common information-theoretic criteria including Bhattacharyya distance, KL-divergence, J-divergence, and mutual information as design metrics. We cast the design problems under a unified optimization framework and devise two algorithms to deal with these problems. We extend the proposed methods to the case of PAR-constrained design. We also propose a novel method for (constrained) transmit code design in non-orthogonal MIMO radars.

Keywords: Detection, Doppler shift, optimization, waveform design, signal-dependent interference.

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Introduction

Radars as well as many other active sensing systems face the simultaneous effects of signal-dependent and independent interferences. In addition, the target Doppler shift is usually unknown at the transmitter. Considering such an ambiguity along with the presence of clutter, and the practical implementation demands (e.g., low PAR codes) make the transmit signal design a challenging task.

The signal (code) design for radar performance improvement has been an active area of research in the last decades; however, the majority of previous works have considered either stationary targets or clutter-free scenarios. The effect of clutter has been considered in early studies for stationary targets, or targets with known Doppler shifts (see e.g. [1]). Several researches consider signal-independent clutter scenarios (see e.g. [2], [3]). The unknown Doppler shift of the target has been taken into account in [3]. In multiple-input multiple-output (MIMO) scenarios, due to complicated expressions for performance analysis, usually information-theoretic criteria are considered as design metrics to guarantee some types of optimality for the obtained signals (see e.g. [4]). Similar to the SISO case, several works deals with clutter-free scenarios [2], [5]. To the best of our knowledge, the reported researches consider unconstrained design.

In this dissertation, we study the problem of radar waveform design for detection performance improvement. As to the SISO systems, we consider the simultaneous effects of the signal-dependent interference and unknown Doppler shift of the target. First, two different design methodologies including *average* and *worst-case* approaches are considered to handle the fact that the Doppler shift of the target is unknown at the transmit side. We propose **Co**nvexification via **Re**parametrization (CoRe) and **C**yclic **A**lgorithm for **D**irect **CO**de **DE**sign (which we call CADCODE) frameworks for tackling the design problems. The extension to the case of PAR-constrained design is also provided. Second, we devise a method for **D**oppl**e**r robu**s**t jo**i**nt **de**sign (which we call DESIDE) of transmit sequence and receive filter (with a similarity constraint). In case of MIMO systems, we derive exact theoretical performance expressions of the optimal detector along with bounds on the performance (for Gaussian interference). Due to complicated forms for the mentioned expressions, we employ several information-theoretic criteria including Bhattacharyya distance, KL-divergence, J-divergence, and MI as metrics for code design (in the presence of clutter). We show that the arising optimization problems can be conveniently dealt with using a unified framework. To tackle the code design problem, two novel methods (which we call Sv-MaMi and Re-MaMi) based on **Ma**jorization-**Mi**nimization (MaMi) technique are devised. We extend the proposed methods to design with PAR constraints and to the case of multiple transmitters (with orthogonal transmission). We also devise another method (called T-MaMi) based on majorization of matrix functions for (constrained) code design in non-orthogonal MIMO radars.

SISO systems

• **Transmit code design for pulsed-radars**

The discrete-time received signal for a pulsed-radar can be expressed as

$$\mathbf{r} = \alpha\, \mathbf{a} \odot \mathbf{p} + \mathbf{a} \odot \mathbf{c} + \mathbf{n} \tag{1}$$

where α is associated with target return (with variance σ^2), $\mathbf{a}$ is the transmit (inter-pulse) code vector, $\mathbf{p} \triangleq [1 \; e^{j\omega} \quad \ldots \quad e^{j(N-1)\omega}]^T$ with ω being the normalized Doppler shift of the target and N being the number of transmitted pulses, $\mathbf{c}$ is the vector corresponding to the clutter component (considering unambiguous-range scatterers), and finally the vector $\mathbf{n}$ represents the signal-independent interferences (with covariance $\mathbf{M}$). For a priori known Doppler shift, the probability of detection (for any given false alarm probability) is monotonically increasing function of $\lambda = \sigma^2(\mathbf{a} \odot \mathbf{p})^H \left(\mathbf{M} + \mathbf{A}\mathbf{C}\mathbf{A}^H\right)^{-1} (\mathbf{a} \odot \mathbf{p})$ with $\mathbf{A} = \mathrm{Diag}(\mathbf{a})$ and $\mathbf{C} = \mathrm{E}\{\mathbf{c}\mathbf{c}^H\}$. We use average and worst-case design approaches to handle the fact that the target Doppler shift ω is usually unknown at the transmit side. In the average design, the average of λ is consider as design metric and hence the design problem can be cast as

$$\max_{\mathbf{A}} \quad \mathrm{tr}\left\{\left(\mathbf{A}^{-1}\mathbf{M}\mathbf{A}^{-H} + \mathbf{C}\right)^{-1} \mathbf{W}\right\} \quad \text{subject to} \quad \mathrm{tr}\left\{\mathbf{A}\mathbf{A}^H\right\} \leq e \tag{2}$$

where $\mathbf{W} = \mathrm{E}\{\mathbf{p}\mathbf{p}^H\}$ and e denotes total available energy. Note that maximization of the considered metric for average design leads to maximization of a lower bound on J-divergence associated with the detection problem for unknown Doppler shift. We devise Convexification via Reparametrization (CoRe) framework to tackle the non-convex design problem (2). Indeed, we let $\mathbf{X} \triangleq \mathbf{A}^H\mathbf{M}^{-1}\mathbf{A}$ such that the design problem can be reformulated as a semidefinite program (SDP).To synthesize code matrix $\mathbf{A}$ from solution $\mathbf{X}$ to the mentioned SDP, we consider the following non-convex optimization problem

$$\min_{\mathbf{A},\mathbf{Q}} \quad \|\mathbf{X}^{1/2}\mathbf{Q} - \mathbf{A}^H\mathbf{M}^{-1/2}\|_F^2 \quad \text{subject to} \quad \mathbf{Q}\mathbf{Q}^H = \mathbf{I}. \tag{3}$$

The above problem can be handled using a cyclic minimization procedure. Regarding average design, we also propose a cyclic algorithm (called CADCODE) to directly obtain a solution to the problem (2). To this end, we consider the following minimization problem for an auxiliary variable $\mathbf{Y}$ and full column-rank matrix $\mathbf{U}$:

$$\begin{cases} \min\limits_{\mathbf{A},\mathbf{Y}} & g(\mathbf{A},\mathbf{Y}) \\ \text{subject to} & \mathbf{Y}^H\mathbf{U} = \mathbf{I}, \operatorname{tr}\{\mathbf{A}^H\mathbf{A}\} \leq e \end{cases} \tag{4}$$

with $g(\mathbf{A},\mathbf{Y}) = \operatorname{tr}\{\mathbf{Y}^H\mathbf{R}\mathbf{Y}\}$ and

$$\mathbf{R} \triangleq \begin{bmatrix} \theta\mathbf{I} & \mathbf{V}^H\mathbf{A}^H \\ \mathbf{A}\mathbf{V} & \mathbf{M} + \mathbf{A}\mathbf{C}\mathbf{A}^H \end{bmatrix} \tag{5}$$

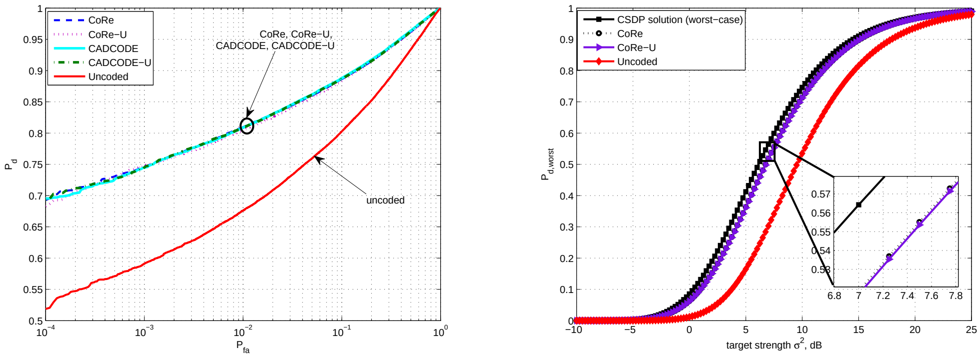
It is shown that cyclic minimization of $g(\mathbf{A},\mathbf{Y})$ leads to an iterative minimization of the original design problem (2).

In worst-case design, the minimum value of λ w.r.t target Doppler shift ω is used as design metric; in other words, the optimization problem for this case is:

$$\max_{\mathbf{A}} \min_{\omega\in\Omega} \quad \operatorname{tr}\left\{\left((\mathbf{A}^H\mathbf{M}^{-1}\mathbf{A})^{-1} + \mathbf{C}\right)^{-1} \mathbf{p}\mathbf{p}^H\right\} \quad \text{subject to} \quad \operatorname{tr}\left\{\mathbf{A}^H\mathbf{A}\right\} \leq e \tag{6}$$

where $\Omega = [\omega_l, \omega_u]$ denotes the desired interval for ω. Note that the worst-case design indirectly assumes that the radar receiver employs a filter bank (tuned on various Doppler shifts) for target detection. The CoRe framework can be used to obtain a solution to the above problem. More precisely, by defining $\mathbf{Z} = \left((\mathbf{A}^H\mathbf{M}^{-1}\mathbf{A})^{-1} + \mathbf{C}\right)^{-1}$, an SDP can be obtained for the worst-case design. A synthesis stage similar to that of average design is used to obtain code matrix $\mathbf{A}$ from solution $\mathbf{Z}$.

Note that both CoRe and CADCODE frameworks can be extended to the case of PAR-constrained design. For CoRe, an arbitrary PAR constraint can be imposed in the synthesis stage. For CADCODE, such a constraint is considered in the constraint set of (4). Such constrained design problems lead to NP-hard problem in general; but solutions to these problems can be obtained via iterative methods. In Fig. 1(a) the ROC of the optimal detector (assuming unknown Doppler shift) is plotted for system using CoRe and CADCODE as well as uncoded system. Fig. 1(b) shows the worst detection probability corresponding to the system using devised methods and uncoded one. For both figures, typical scenarios are assumed.



(a) (b)

Fig. 1. (a) average design: ROC of the coded system using proposed methods (with and without PAR=1 constraint) and uncoded system, (b) worst-case design: the worst detection probability corresponding to the proposed method and uncoded system.

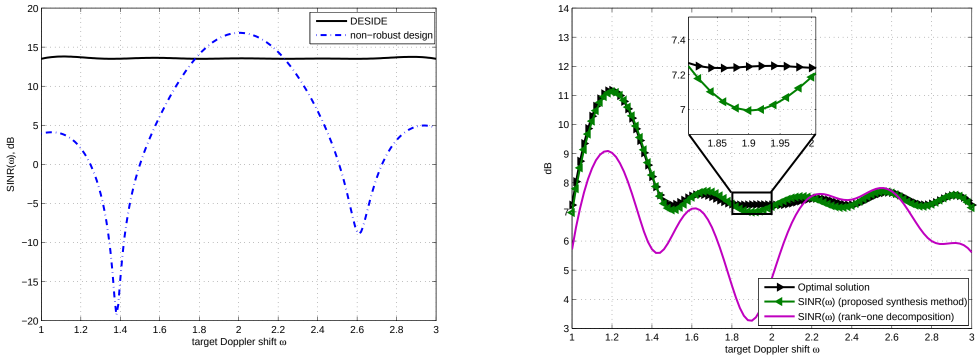
• Doppler robust joint design of transmit code and receive filter

Herein we assume that the radar receiver consists of a filter $\mathbf{w}$ and aim to obtain Doppler robust pair of transmit code and receive filter. We consider the output SINR of the receive filter as the performance measure of the system and impose a similarity constraint on the transmit code. The design problem can be cast as

$$\mathcal{P}\begin{cases}\max\limits_{\mathbf{a},\mathbf{w}}\ \min\limits_{\omega\in\Omega} & \dfrac{\left|\mathbf{w}^H\left(\mathbf{a}\odot\mathbf{p}(\omega)\right)\right|^2}{\mathbf{w}^H\boldsymbol{\Sigma}_{\mathbf{c}}\left(\mathbf{a}\right)\mathbf{w}+\mathbf{w}^H\mathbf{M}\mathbf{w}}\\ \text{subject to} & \|\mathbf{a}\|^2=e,\ \|\mathbf{a}-\mathbf{a}_0\|^2\leq\delta\end{cases} \tag{7}$$

where $\boldsymbol{\Sigma}_{\mathbf{c}}\left(\mathbf{a}\right)$ denotes the covariance matrix of the clutter component and δ rules the similarity region. We devise a method for **D**opple**r** robust jo**i**nt **de**sign of transmit sequence and receive filter (which we call DESIDE). To this end, we rewrite the objective function w.r.t $\mathbf{X}=\mathbf{a}\mathbf{a}^H$ and $\mathbf{W}=\mathbf{w}\mathbf{w}^H$. Applying DESIDE to the design problem consists of two stages. At the first stage, a relaxed version (i.e., with no rank one constraints on $\mathbf{X}$ and $\mathbf{W}$) of the design problem (7) is tackled via a cyclic minimization approach. We show that each iteration of the cyclic algorithm can be handled by solving two SDPs; for example, the SDP for given transmit code is:

$$\mathcal{SDP}_W\begin{cases}\max\limits_{\mathbf{W},\mathbf{Z}_1',\mathbf{Z}_2',\breve{t}} & \breve{t}\\ \text{subject to} & \operatorname{tr}\left\{\left(\boldsymbol{\Sigma}_{\mathbf{c}}\left(\mathbf{X}\right)+\mathbf{M}\right)\mathbf{W}\right\}=1\\ & \mathbf{z}'=\breve{t}\mathbf{e}_1+\mathbf{F}_1^H\left(\operatorname{diag}(\mathbf{F}_1\mathbf{Z}_1'\mathbf{F}_1^H)+\mathbf{q}\odot\operatorname{diag}(\mathbf{F}_2\mathbf{Z}_2'\mathbf{F}_2^H)\right)\\ & \mathbf{W}\succeq\mathbf{0},\mathbf{Z}_1'\succeq\mathbf{0},\mathbf{Z}_2'\succeq\mathbf{0}\end{cases} \tag{8}$$



(a) (b)

Fig. 2. (a) a typical comparison of SINR for a pair obtained by DESIDE with the result of non-robust design, (b) comparison of the performance of the proposed synthesis algorithms with a rank-one decomposition method.

where $\mathbf{Z}_1'$, $\mathbf{Z}_2'$, and $\breve{t}$ are auxiliary variables and $\mathbf{F}_1$ as well as $\mathbf{F}_2$ are associated with Fourier matrix. At the second stage, we propose novel synthesis algorithms to obtain Doppler robust pairs of transmit codes and receive filters. The idea of the synthesis stage is to obtain transmit code/receive filter such that they approximate well the behavior of the solution to the relaxed problem w.r.t Doppler shift. The synthesis stage is cast as non-convex optimization problems which are handled via other cyclic methods. Note that DESIDE can be applied to both inter-pulse and intra-pulse coded systems. Fig. 2(a)-(b) show SINR obtained using DESIDE and the result of applying the proposed synthesis algorithm for typical scenarios.

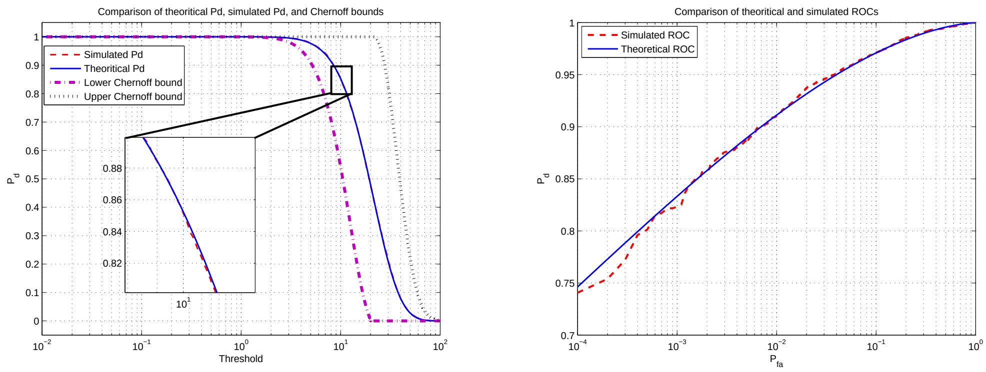
MIMO systems

- **Performance analysis of the optimal detector**

Consider the following detection problem for an MIMO radar:

$$\begin{cases} \mathrm{H}_0: & \mathbf{r}_k = \mathbf{n}_k \\ \mathrm{H}_1: & \mathbf{r}_k = \mathbf{A}\boldsymbol{\alpha}_k + \mathbf{n}_k \end{cases}, k = 1, 2, ..., N_r \tag{9}$$

where $\mathbf{r}_k$ denotes the received signal at the k^{th} receiver, $\mathbf{A}$ is a matrix that its columns correspond to the transmission code from various transmit antennas, and $\boldsymbol{\alpha}_k$ contains target scattering coefficients associated with different transmitters at the k^{th} receiver. Note that in the above, it is assumed that the transmission signals from various transmitters are not orthogonal; but the results can be straightforwardly applied to MIMO radars with orthogonal transmission. We show the optimal detector (for Gaussian interference) can be written as weighted sums



(a) (b)

Fig. 3. (a) comparison of the theoretical and simulated detection probability along with associated lower and upper bounds, (b) comparison of the obtained ROC using theoretical expressions and simulated ROC.

of independent chi-square random variables in both hypotheses.We prove a lemma in which the statistical distribution of the weighted sum of independent chi-square random variables is obtained. Using this lemma, the probability of detection and false alarm of the optimal detector are available. As the performance expressions and detector behavior are complicated, we also employ Chernoff bound to derive lower and upper bounds on the probability of detection and false alarm. Comparisons of performance metrics obtained by theoretical expressions and simulated method are provided in Fig. 3.

- **Code design for MIMO radars with orthogonal transmission**

We consider a multi-static pulsed-radar with one transmitter and N_r widely separated receive antennas. The discrete-time signal corresponding to a stationary target (and a certain radar cell) for the k^{th} receiver can be described as

$$\mathbf{r}_k \triangleq \mathbf{s}_k + \mathbf{c}_k + \mathbf{n}_k = \alpha_k \mathbf{a} + \widetilde{\rho}_k \mathbf{a} + \mathbf{n}_k \tag{10}$$

where $\widetilde{\rho}_k$ is a Gaussian random variable (with variance $\sigma^2_{\mathrm{c},k}$) associated with clutter component. The optimal detector possesses complicated performance expressions (that can be obtained from the results of the previous item); and hence we resort to information-theoretic criteria including Bhattacharyya distance $\mathcal{B}$, KL-divergence $\mathcal{D}$, J-divergence $\mathcal{J}$, and MI $\mathcal{M}$ as design metrics. We show that design problems corresponding to various metrics can be cast under a unified optimization framework. Let $\lambda_k \triangleq \sigma_k^2 \mathbf{a}^H (\sigma_{\mathrm{c},k}^2 \mathbf{a}\mathbf{a}^H + \mathbf{M}_k)^{-1}\mathbf{a}$. We have that

$$\begin{cases} \max\limits_{\mathbf{a},\lambda_k} & \sum_{k=1}^{N_r} \left[f_{\mathcal{I}}(\lambda_k) + g_{\mathcal{I}}(\lambda_k) \right] \\ \text{subject to} & \lambda_k = \sigma_k^2 \mathbf{a}^H (\sigma_{\mathrm{c},k}^2 \mathbf{a}\mathbf{a}^H + \mathbf{M}_k)^{-1}\mathbf{a} \\ & \|\mathbf{a}\|^2 \leqslant e \end{cases} \tag{11}$$

where $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}, \mathcal{J}, \mathcal{M}\}$, and $f_{\mathcal{I}}(.)$ and $g_{\mathcal{I}}(.)$ are concave and convex functions for any $\mathcal{I}$, respectively. More precisely,

$$\begin{cases} f_{\mathcal{B}}(\lambda_k) = \log(1 + 0.5\lambda_k), & g_{\mathcal{B}}(\lambda_k) = -\frac{1}{2}\log(1+\lambda_k), \\ f_{\mathcal{D}}(\lambda_k) = \log(1+\lambda_k), & g_{\mathcal{D}}(\lambda_k) = \frac{1}{1+\lambda_k} - 1, \\ f_{\mathcal{J}}(\lambda_k) = 0, & g_{\mathcal{J}}(\lambda_k) = \frac{\lambda_k^2}{1+\lambda_k}, \\ f_{\mathcal{M}}(\lambda_k) = \log(1+\lambda_k), & g_{\mathcal{M}}(\lambda_k) = 0. \end{cases}$$

We propose two general algorithms based on MaMi to tackle the optimization problems in (11). The first algorithm (which we call Sv-MaMi) uses successive (linear and quadratic) majorizations whereas the second one (which is called Re-MaMi) is based on a relaxation, MaMi technique, and a synthesis stage.

For Sv-MaMi, we show that the $(l+1)^{th}$ iteration of this algorithm can be handled by solving the following convex quadratically-constrained quadratic program (QCQP):

$$\begin{cases} \min\limits_{\mathbf{a}} & \mathbf{a}^H \left(\sum_{k=1}^{N_r} \phi_{k,\mathcal{I}}^{(l)} \mathbf{M}_k^{-1} \right) \mathbf{a} - \Re\left(\left(\sum_{k=1}^{N_r} \mathbf{d}_{k,\mathcal{I}}^{(l)} \right)^H \mathbf{a} \right) \\ \text{subject to} & \|\mathbf{a}\|^2 \leqslant e \end{cases} \tag{12}$$

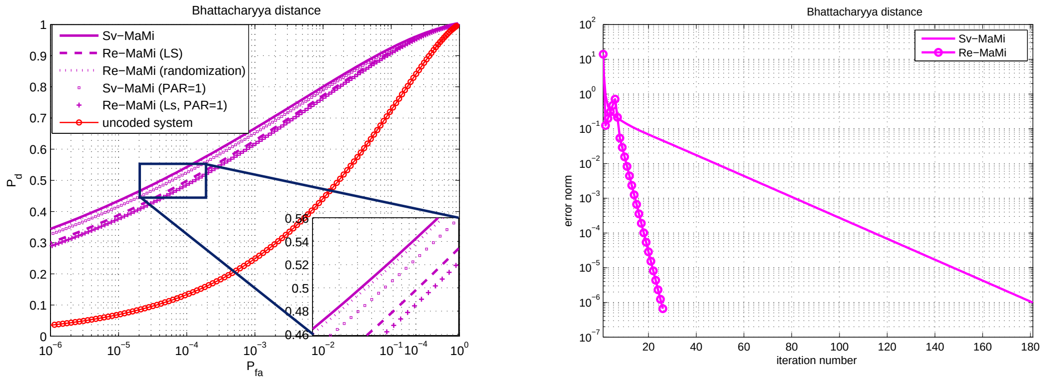
where the positive constant $\{\phi_{k,\mathcal{I}}^{(l)}\}$ and the vectors $\{\mathbf{d}_{k,\mathcal{I}}^{(l)}\}$ depend on $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}, \mathcal{J}, \mathcal{M}\}$.

In Re-MaMi, we obtain the following convex problem for the $(l+1)^{th}$ iteration:

$$\begin{cases} \max\limits_{\mathbf{A}\succeq \mathbf{0}} & \sum_{k=1}^{N_r} \left[f_{\mathcal{I}}\left(N\gamma_k - \gamma_k \operatorname{tr}\{(\mathbf{M}_k + \beta_k \mathbf{A})^{-1}\mathbf{M}_k\} \right) + h_{k,\mathcal{I}}^{(l)}(\mathbf{A}) \right] \\ \text{subject to} & \operatorname{tr}\{\mathbf{A}\} \leqslant e \end{cases} \tag{13}$$

where $\mathbf{A} = \mathbf{a}\mathbf{a}^H$ and $h_{k,\mathcal{I}}^{(l)}(\mathbf{A})$ denotes a concave function of $\mathbf{A}$, for $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}, \mathcal{J}, \mathcal{M}\}$. The Re-MaMi employs the least-squares or the randomization technique to synthesize optimized transmit code $\mathbf{a}$ from the obtained solution $\mathbf{A}$.

Note that the proposed algorithms can be extended to the case of multiple transmitters with orthogonal transmission. Moreover, Sv-MaMi and Re-MaMi can also be modified to tackle PAR-constrained code design. Fig. 4(a) depicts ROCs of the coded system using proposed methods



(a) (b)

Fig. 4. (a) ROC of the coded system for Sv-MaMi, Re-MaMi (both constrained and unconstrained) as well as uncoded system corresponding to Bhatacharyya distance, (b) the error norm versus iteration number for Sv-MaMi and Re-MaMi ($\mathcal{B}$).

(with $\mathcal{B}$) and uncoded one for a typical situation. The part (b) of this figure illustrates the error norm[1] versus iteration number. Note that from a computational point of view, Sv-MaMi is preferable for $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}\}$.

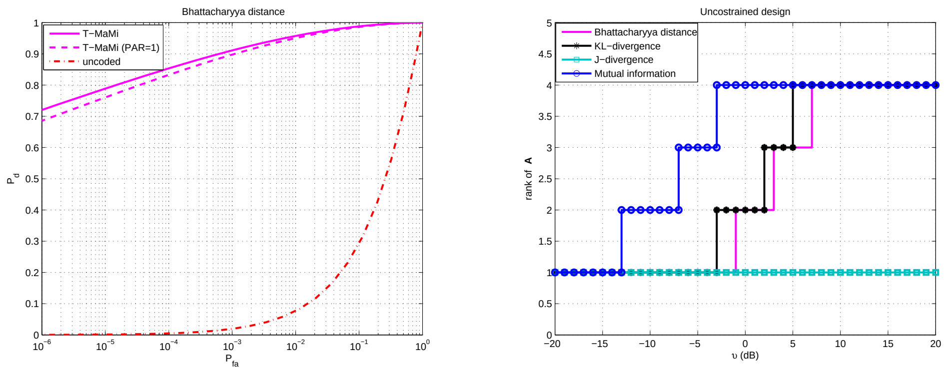
- **Code design for non-orthogonal MIMO radars**

For non-orthogonal MIMO radars we use the signal model provided in the related references; indeed, in these systems, the detection problem can be cast as (9). We consider a general case for which $\mathbf{R}_k = \mathrm{E}\{\boldsymbol{\alpha}_k \boldsymbol{\alpha}_k^H\}$ and $\mathbf{M}_k = \mathrm{E}\{\mathbf{n}_k \mathbf{n}_k^H\}$. Again, the performance expressions of the optimal detector is complicated for code design and so we employ information-theoretic criteria as design metrics. Let $\mathbf{H}_k \triangleq \mathbf{M}_k^{-1/2} \mathbf{A} \mathbf{R}_k \mathbf{A}^H \mathbf{M}_k^{-1/2}$. The design problems corresponding to the criteria can be cast under a unified optimization framework:

$$\begin{cases} \max\limits_{\mathbf{A} \in \mathcal{C}, \mathbf{H}_k} & \sum_{k=1}^{N_r} \left[f_{\mathcal{I}}(\mathbf{H}_k) + g_{\mathcal{I}}(\mathbf{H}_k) \right] \\ \text{subject to} & \mathbf{H}_k = \mathbf{M}_k^{-1/2} \mathbf{A} \mathbf{R}_k \mathbf{A}^H \mathbf{M}_k^{-1/2} \end{cases} \tag{14}$$

where $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}, \mathcal{J}, \mathcal{M}\}$. Herein, $\mathcal{C}$ represents the considered constraint set for the design; e.g.,

[1]The error norm for Sv-MaMi and Re-MaMi is defined as $\|\mathbf{a}^{(l+1)} - \mathbf{a}^{(l)}\|$ and $\|\mathbf{A}^{(l+1)} - \mathbf{A}^{(l)}\|_F$, respectively.



(a) (b)

Fig. 5. (a) ROC of the coded system using T-MaMi (constrained and unconstrained) as well as uncoded system for $\mathcal{B}$, (b) comparison of the rank of the optimal code matrix $\mathbf{A}$ versus υ for various criteria (unconstrained).

energy or unimodularity constraint. In addition, $f, g : \mathcal{S}_+^N \rightarrow \mathbb{R}$ and we have

$$\begin{cases} f_{\mathcal{B}}(\mathbf{H}_k) = \log\det\left(\mathbf{I} + 0.5\mathbf{H}_k\right), & g_{\mathcal{B}}(\mathbf{H}_k) = -0.5\log\det\left(\mathbf{I} + \mathbf{H}_k\right), \\ f_{\mathcal{D}}(\mathbf{H}_k) = \log\det\left(\mathbf{I} + \mathbf{H}_k\right), & g_{\mathcal{D}}(\mathbf{H}_k) = \mathrm{tr}\{(\mathbf{I} + \mathbf{H}_k)^{-1} - \mathbf{I}\}, \\ f_{\mathcal{J}}(\mathbf{H}_k) = 0, & g_{\mathcal{J}}(\mathbf{H}_k) = \mathrm{tr}\{\mathbf{H}_k - \mathbf{H}_k(\mathbf{I} + \mathbf{H}_k)^{-1}\}, \\ f_{\mathcal{M}}(\mathbf{H}_k) = \log\det\left(\mathbf{I} + \mathbf{H}_k\right), & g_{\mathcal{M}}(\mathbf{H}_k) = 0. \end{cases} \tag{15}$$

It is observed that $f_{\mathcal{I}}(.)$ and $g_{\mathcal{I}}(.)$ are concave and convex functions of $\mathbf{H}_k$ (for any $\mathcal{I}$), respectively. We devise a general method (which we call T-MaMi) to deal with the stated problems in (14). T-MaMi is based on rewriting the objective functions as convex functions of suitable variables and then majorization of those matrix functions. We prove that the $(l+1)^{th}$ iteration of T-MaMi can be handled via solving the following problem

$$\begin{cases} \max\limits_{\widetilde{\mathbf{a}}} & \widetilde{\mathbf{a}}^H \mathbf{Q}_{\mathcal{I}}^{(l)} \widetilde{\mathbf{a}} + 2\Re\left((\mathbf{q}_{\mathcal{I}}^{(l)})^H \widetilde{\mathbf{a}}\right) \\ \text{subject to} & \widetilde{\mathbf{a}} \in \mathcal{C}_v \end{cases} \tag{16}$$

where $\widetilde{\mathbf{a}} = \mathrm{vec}(\mathbf{A})$, matrix $\mathbf{Q}_{\mathcal{I}}^{(l)}$ as well as vector $\mathbf{q}_{\mathcal{I}}^{(l)}$ depend on $\mathcal{I} \in \{\mathcal{B}, \mathcal{D}, \mathcal{J}, \mathcal{M}\}$, and $\mathcal{C}_v$ is a set associated with $\mathcal{C}$. We propose algorithms to obtain solutions to the problem in (16) for various constraint sets. ROCs of the coded system with T-MaMi and uncoded one (for $\mathcal{B}$ and a typical situation) are plotted in Fig. 5 (a). Fig. 5 (b) shows the rank of the optimal code matrix $\mathbf{A}$ versus an SNR indicator (υ) for various criteria.

Conclusion

The problem of waveform design was studied for improvement of detection performance. The effect of the clutter and the fact that Doppler shift of targets are often unknown at the transmit side are considered for SISO radars. We devised several methods to tackle the non-convex and in some cases NP-hard design problems and extended the proposed algorithms to the case of constrained design (under PAR or similarity constraint). In case of MIMO systems, the exact theoretical performance expressions of the optimal detector along with related bounds were derived. Due to complicated forms of the expressions, the most common information-theoretic criteria including Bhattacharyya distance, KL-divergence, J-divergence, and mutual information were used as design metrics. We cast the design problems under a unified optimization framework and devised two algorithms to deal with these problems. We also provide the extension to the case of PAR-constrained design. Next, we suggested a novel method for (unimodular) transmit code design in non-orthogonal MIMO radars.

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